RADIATIVE VIEW FACTORS

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VIEW FACTOR DEFINITION

The view factor $F_{12}$ is the fraction of energy exiting an isothermal, opaque, and diffuse surface 1 (by emission or reflection), that directly impinges on surface 2 (to be absorbed, reflected, or transmitted). View factors depend only on geometry. Some view factors having an analytical expression are compiled below. We will use the subindices in $F_{12}$ without a separator when only a few single view-factors are concerned, although more explicit versions, like $F_{1,2}$, or even better, $F_{1\rightarrow2}$, could be used.

From the above definition of view factors, we get the explicit geometrical dependence as follows. Consider two infinitesimal surface patches, $dA_1$ and $dA_2$ (Fig. 1), in arbitrary position and orientation, defined by their separation distance $r_{12}$, and their respective tilting relative to the line of centres, $\beta_1$ and $\beta_2$, with $0 \leq \beta_1 \leq \pi/2$ and $0 \leq \beta_2 \leq \pi/2$ (i.e. seeing each other). The expression for $dF_{12}$ (we used the differential symbol ‘d’ to match infinitesimal orders of magnitude, since the fraction of the radiation from surface 1 that reaches surface 2 is proportional to $dA_2$), in terms of these geometrical parameters is as follows. The radiation power intercepted by surface $dA_2$ coming directly from a diffuse surface $dA_1$ is the product of its
Radiative view factors $L_1 = M_1 / \pi$, times its perpendicular area $dA_{1\perp}$, times the solid angle subtended by $dA_2$, $d\Omega_{12}$; i.e. $d^2\Phi_{12} = L_1 dA_{1\perp} d\Omega_{12} = L_1 (dA_1 \cos(\beta_1)) dA_2 \cos(\beta_2) / r_{12}^2$. Thence:

$$dF_{12} \equiv \frac{d^2\Phi_{12}}{M_1 dA_1} = \frac{L_1 dA_{1\perp} \cos(\beta_1)}{M_1 dA_1} = \frac{\cos(\beta_1)}{\pi} d\Omega_{12} = \frac{\cos(\beta_1)}{\pi} \frac{dA_2 \cos(\beta_2)}{r_{12}^2} = \frac{\cos(\beta_1) \cos(\beta_2)}{\pi r_{12}^2} dA_2$$ (1)

**Fig. 1. Geometry for view-factor definition.**

When finite surfaces are involved, computing view factors is just a problem of mathematical integration (not a trivial one, except in simple cases). Notice that the view factor from a patch $dA_1$ to a finite surface $A_2$, is just the sum of elementary terms, whereas for a finite source, $A_1$, the total view factor, being a fraction, is the average of the elementary terms, i.e. the view factor between finite surfaces $A_1$ and $A_2$ is:

$$F_{12} = \frac{1}{A_1} \int_A \left( \int_{A_2} \frac{\cos(\beta_1) \cos(\beta_2)}{\pi r_{12}^2} dA_2 \right) dA_1$$ (2)

Recall that the emitting surface (exiting, in general) must be isothermal, opaque, and Lambertian (a perfect diffuser for emission and reflection), and, to apply view-factor algebra, all surfaces must be isothermal, opaque, and Lambertian. Finally notice that $F_{12}$ is proportional to $A_2$ but not to $A_1$.

**View factor algebra**

When considering all the surfaces under sight from a given one (let the enclosure have $N$ different surfaces, all opaque, isothermal, and diffuse), several general relations can be established among the $N^2$ possible view factors $F_{ij}$, what is known as view factor algebra:

- **Bounding.** View factors are bounded to $0 \leq F_{ij} \leq 1$ by definition (the view factor $F_{ij}$ is the fraction of energy exiting surface $i$, that impinges on surface $j$).
- **Closeness.** Summing up all view factors from a given surface in an enclosure, including the possible self-view factor for concave surfaces, $\sum_j F_{ij} = 1$, because the same amount of radiation emitted by a surface must be absorbed.
- **Reciprocity.** Noticing from the above equation that $dA_i dF_{ij} = dA_j dF_{ji} = (\cos(\beta_1) \cos(\beta_2) / (\pi r_{12}^2)) dA_i dA_j$, it is deduced that $A_i F_{ij} = A_j F_{ji}$.
- **Distribution.** When two target surfaces ($j$ and $k$) are considered at once, $F_{i,j,k} = F_{ij} + F_{ik}$, based on area additivity in the definition.
• Composition. Based on reciprocity and distribution, when two source areas are considered together, \( F_{i,j,k} = \left( A_i F_{ik} + A_j F_{jk} \right) / (A_i + A_j) \).

One should stress the importance of properly identifying the surfaces at work; e.g. the area of a square plate of 1 m in side may be 1 m² or 2 m², depending on our considering one face or the two faces. Notice that the view factor from a plate 1 to a plate 2 is the same if we are considering only the frontal face of 2 or its two faces, but the view factor from a plate 1 to a plate 2 halves if we are considering the two faces of 1, relative to only taking its frontal face.

For an enclosure formed by \( N \) surfaces, there are \( N^2 \) view factors (each surface with all the others and itself). But only \( N(N-1)/2 \) of them are independent, since another \( N(N-1)/2 \) can be deduced from reciprocity relations, and \( N \) more by closeness relations. For instance, for a 3-surface enclosure, we can define 9 possible view factors, 3 of which must be found independently, another 3 can be obtained from \( A_i F_{ij} = A_j F_{ji} \), and the remaining 3 by \( \sum F_{ij} = 1 \).

**View factors with two-dimensional objects**

Consider two infinitesimal surface patches, \( dA_1 \) and \( dA_2 \), each one on an infinitesimal long parallel strip as shown in Fig. 2. The view factor \( dF_{12} \) is given by (1), where the distance between centres, \( r_{12} \), and the angles \( \beta_1 \) and \( \beta_2 \) between the line of centres and the respective normal are depicted in the 3D view in Fig. 2a, but we want to put them in terms of the 2D parameters shown in Fig. 2b (the minimum distance \( a = \sqrt{x^2 + y^2} \), and the \( \beta_1 \) and \( \beta_2 \) angles when \( z=0, \beta_{10} \) and \( \beta_{20} \)), and the depth \( z \) of the \( dA_2 \) location. The relationship are: \( r_{12} = \sqrt{x^2 + y^2 + z^2} = \sqrt{a^2 + z^2} \), \( \cos \beta_1 = \cos \beta_{10} \cos \gamma \), with \( \cos \beta_1 = y/r_{12} = (y/a)(a/r_{12}) \), \( \cos \beta_{10} = y/a \), \( \cos \gamma = a/r_{12} \), and \( \cos \beta_2 = \cos \beta_{20} \cos \gamma \), therefore, between the two patches:

\[
dF_{12} = \frac{\cos(\beta_1)\cos(\beta_2)}{\pi r_{12}^2} dA_2 = \frac{a^2 \cos(\beta_{10})\cos(\beta_{20})}{\pi r_{12}^4} dA_2 = \frac{a^2 \cos(\beta_{10})\cos(\beta_{20})}{\pi(a^2 + z^2)^2} dA_2
\]

(3)

Fig. 2. Geometry for view-factor between two patches in parallel strips: a 3D sketch, b) profile view.

Expression (3) can be reformulated in many different ways; e.g. by setting \( d^2A_2 = dwdz \), where the ‘\( d^2 \)’ notation is used to match differential orders and \( dw \) is the width of the strip, and using the relation \( ad\beta_{10} = \cos \beta_{20} dw \). However, what we want is to compute the view factor from the patch \( dA_1 \) to the whole strip from \( z=-\infty \) to \( z=+\infty \), what is achieved by integration of (3) in \( z \):

Radiative view factors
\[
\frac{d^2 F_{12}}{d\beta_{10} d\beta_{20}} = \frac{a^2 \cos(\beta_{10}) \cos(\beta_{20})}{\pi (a^2 + z^2)^2} \quad \text{d}w_d \quad \rightarrow \quad dF_{12} = \int_{-\infty}^{\infty} d^2 F_{12} dz = \frac{\cos(\beta_{10}) \cos(\beta_{20})}{2a} \quad \text{d}w = \frac{\cos(\beta_{10})}{2} d\beta_{10} \quad (4)
\]

For instance, approximating differentials by small finite quantities, the fraction of radiation exiting a patch of \(A_1=1\) cm\(^2\), that impinges on a parallel and frontal strip \((\beta_{10}=\beta_{20}=0)\) of width \(w=1\) cm separated a distance \(a=1\) m apart is \(F_{12}=w/(2a)=0.01/(2\cdot1)=0.005\), i.e. a 0.5 %. It is stressed again that the exponent in the differential operator ‘d’ is used for consistency in infinitesimal order.

Now we want to know the view factor \(dF_{12}\) from an infinite strip \(dA_1\) (of area per unit length \(dw_1\)) to an infinite strip \(dA_2\) (of area per unit length \(dw_2\)), with the geometry presented in Fig. 2. It is clear from the infinite extent of strip \(dA_2\) that any patch \(d^2 A_1=dw_1dz_1\) has the same view factor to the strip \(dA_2\), so that the average coincides with this constant value and, consequently, the view factor between the two strips is precisely given by (4); i.e. following the example presented above, the fraction of radiation exiting a long strip of \(w_1=1\) cm width, that impinges on a parallel and frontal strip \((\beta_{10}=\beta_{20}=0)\) of width \(w_2=1\) cm separated a distance \(a=1\) m apart is \(F_{12}=w_2/(2a)=0.01/(2\cdot1)=0.005\), i.e. a 0.5 %.

Notice the difference in view factors between the two strips and the two patches in the same position as in Fig. 2b: using \(dA_1\) and \(dA_2\) in both cases, the latter (3D case) is given by the general expression (1), which takes the form \(dF_{12}=\cos(\beta_{10})\cos(\beta_{20})dA_2/\pi a^2\), whereas in the two-strip case (2D), it is \(dF_{12}=\cos(\beta_{10})\cos(\beta_{20})dA_2/(2a)^2\), where \(A_2\) has now units of length (width of the strip).

**Very-long triangular enclosure**

Consider a long duct with the triangular cross section shown in Fig. 3. We may compute the view factor \(F_{12}\) from face 1 to face 2 (inside the duct) by double integration of the view factor from a strip of width \(dw_1\) in \(L_1\) to strip \(dw_2\) in \(L_2\); e.g. using the strip-to-strip view factor (4), the strip to finite band view factor is \(F_{12}=[\cos(\beta_{10})d(\beta_{10})/(\sin(\beta_{10})-\sin(\beta_{10}))/2\), where \(\beta_{10}\) start and \(\beta_{10}\) end are the angular start and end directions subtended by the finite band 2 from infinitesimal strip 1. Let see an example to be more explicit.

![Fig. 3. Triangular enclosure.](image)

**Example 1.** Find the view factor \(F_{12}\) from \(L_1\) to \(L_2\) for \(\phi=90^\circ\) in Fig. 3, i.e. between two long perpendicular strips touching, by integration of the case for infinitesimal strips (4).
Referring to Fig. 2b, the view factor from a generic infinitesimal strip at $x$ (from the edge), to the whole band at 2, becomes $F_{12} = (\sin \beta_{10\text{end}} - \sin \beta_{10\text{start}})/2 = (1 - \sqrt{x^2 + L_2^2})/2$, and, upon integration on $x$, we get the view factor from finite band 1 to finite band 2: $F_{12} = (1/L)(1 - \sqrt{x^2 + L_2^2})dx/2 = \left(L_1 + L_2 - \sqrt{L_1^2 + L_2^2}\right)/(2L_1)$.

But it is not necessary to carry out integrations because all view factors in such a simple triangular enclosure can be found by simple application of the view-factor algebra presented above. To demonstrate it, we first establish the closure relation $\sum F_{ij} = 1$ at each of the three nodes, noticing that for non-concave surfaces $F_{ii} = 0$; then we multiply by their respective areas (in our case $L_1$, $L_2$, $L_3$, by unit depth); next, we apply some reciprocity relations, and finally perform the combination of equations as stated below:

$$0 + F_{12} + F_{13} = 1 \quad \rightarrow \quad L_1 F_{12} + L_1 F_{13} = L_1$$

(5)

$$F_{21} + 0 + F_{23} = 1 \quad \rightarrow \quad L_2 F_{21} + L_2 F_{23} = L_2$$

(6)

$$F_{31} + F_{32} + 0 = 1 \quad \rightarrow \quad L_3 F_{31} + L_3 F_{32} = L_3$$

(7)

(5)+(6)-(7) $\Rightarrow$ $2L_1 F_{12} = L_1 + L_2 - L_3 \quad \rightarrow \quad F_{12} = \frac{L_1 + L_2 - L_3}{2L_1}$

(8)

We see how easy it is now to recover the result for perpendicular bands of width $L_1$ and $L_2$ solved in Example 1, $F_{12} = \left(L_1 + L_2 - \sqrt{L_1^2 + L_2^2}\right)/(2L_1)$; e.g. the view factor between equal perpendicular bands is $F_{12} = \left(2 - \sqrt{2}\right)/2 = 0.293$, i.e. 29% of the energy diffusively outgoing a long strip will directly reach an equal strip perpendicular and hinged to the former, with the remaining 71% being directed to the other side 3 (lost towards the environment if $L_3$ is just an opening).

Even though we have implicitly assumed straight-line cross-sections (Fig. 3), the result (8) applies to convex triangles too (we only required $F_{ii} = 0$), using the real curvilinear lengths instead of the straight distances. As for concave bands, the best is to apply (8) to the imaginary straight-line triangle, and afterwards solve for the trivial enclosure of the real concave shape and its corresponding virtual straight-line.

Example 2. Find the view factor $F_{12}$ from $L_1$ to $L_2$ for $\phi = 90^\circ$ in Fig. 3, i.e. between two long perpendicular strips touching, by integration of the case for infinitesimal strips (4).

Sol. From Example 1 applied to equal perpendicular straight strips, $F_{12} = \left(L_1 + L_2 + \sqrt{L_1^2 + L_2^2}\right)/(2L_1)$, where $L_1 = L_2$ are the diametrical strips to $L_1$ and $L_2$ (dashed in Fig. E2), which we can take as unit length; the enclose is completed with $F_{13} = 1 - F_{12} = 0.707$. Let consider now the cylindrical surfaces; first, it is clear that the radiation arriving at 2’ (from 1’) will arrive at 2 also, i.e. $F_{12} = F_{12}$; second, it is obvious that $F_{11} = 1$, i.e. all radiation outgoing downwards from the planar strip 1’ will go to 1, and, by the reciprocity relation, $F_{11} = A_1 F_{11}/A_1 = 1 - F_{11}/(\pi/2) = 2/\pi = 0.637$, and hence $F_{11} = 1 - F_{11} = 1 - 2/\pi = 0.363$. In summary, from the radiation cast by hemicylinder 1, a fraction $F_{11} = 36\%$ goes against the same concave surface, and, from the remaining $F_{11} = 64\%$ Radiative view factors
that goes upwards, 29% of it goes towards the right (surface 2), and the 71% remaining impinging on surface 3; in consequence, $F_{12} = F_{11}F_{12} = 0.637 \cdot 0.293 = \left(2 - \sqrt{2}\right) / \pi = 0.186$\rightarrow 0.186, $F_{13} = F_{11}F_{13} = 0.637 \cdot 0.707 = \sqrt{2} / \pi = 0.450$, and $F_{11} = 1 - 2 / \pi = 0.363$ (check: 0.363 + 0.186 + 0.450 = 1).

Fig. E2. Sketch to deduce the view factor $F_{12}$ between two long hemicylinders (1) and (2).

Now we generalise this algebraic method of computing view factors in two-dimensional geometries to non-contact surfaces.

**The crossed string method**

For any two non-touching infinitely-long bands, 1 and 2 (Fig. 4), one can also find all the view factors from simple algebraic relations as in the triangular enclosure before, extending the result (8) to what is known as crossed-string method:

$$F_{12} = \frac{L_4 + L_5 - L_1 - L_6}{2L_1} = \frac{\sum \text{crossed strings} - \sum \text{uncrossed strings}}{2 \times (\text{source string})} \tag{9}$$

Fig. 4. Sketch used to deduce $F_{12}$ in the general case of two infinitely long bands.

The result (9) is deduced by applying the triangular relation (8) to triangle 1–3–4 (shadowed in Fig. 4) and triangle 1–5–6, plus the closure relation to the quadrilateral 1–3–2–6 ($F_{13} + F_{12} + F_{16} = 1$), namely:

$$F_{13} = \frac{L_3 + L_5 - L_4}{2L_1} \quad \rightarrow \quad F_{12} = 1 - F_{12} - F_{12} = \frac{L_4 + L_5 - L_1 - L_6}{2L_1} \tag{10}$$

This procedure to compute view factors in two-dimensional configurations (the crossed-string method), was first developed by H.C. Hottel in the 1950s. The extension to non-planar surfaces 1 and 2 is as already presented for triangular enclosures. A further extension is possible to cases where there are obstacles (two-dimensional, of course) partially protruding into sides 3 and/or 6 in the quadrilateral.
1–3–2–6 (Fig. 4); it suffices to account for the real curvilinear length of each string when stretched over the obstacles, as shown in the following example.

Example 3. Find the view factor between two long parallel cylinders of equal radii \( R \), separated a distance \( 2\sqrt{2} R \) between centres, using the crossed-string method.

Sol. With this clever separation, angle \( \theta \) in Fig. E1 happens to be \( \theta=\pi/4 \) (45º), making calculations simpler. We get \( F_{12} \) from (10) by substituting \( L_1=2\pi R \) (the source cylinder), \( L_4 \) and \( L_5 \) (the crossing strings) each by the length \( abcde \), and \( L_3 \) and \( L_6 \) (the non-crossing strings) each by \( 2\sqrt{2} R \) between. The length \( abcde \) is composed of arc \( ab \), segment \( bc \), and so on, which in our special case is \( ab=R\theta=(\pi/4)R \), \( bc=R \), and \( abcde=2(ab+cd)=(\pi/2)R+2R \), and finally \( F_{12}=(L_4+L_5-L_3-L_6)/(2L_1)=(2abcde-4\sqrt{2} R)/(4\pi R)=(\pi R+4R-4\sqrt{2} R)/(4\pi R)=1/4+(1-\sqrt{2})/\pi \approx 0.12 \), as can be checked with the general expression for cylinders in the compilation following.

Fig. E1. Sketch used to deduce \( F_{12} \) between two infinitely long parallel cylinders.

View factor with an infinitesimal surface: the unit-sphere and the hemicube methods

The view factor from an infinitesimal surface \( dA_1 \) to a finite surface \( A_2 \), \( F_{12}=\frac{1}{\pi}\cos(\beta_1)d\Omega_{12}/\pi \), shows that it is the same for any surface subtending a solid angle \( \Omega_{12} \) in the direction \( \beta_1 \). Hence, a convenient way to compute \( F_{12} \), first proposed by Nusselt in 1928, is to find the conical projection of \( A_2 \) on a sphere of arbitrary radius \( r \) centred on \( dA_1 \) (this projection has the same \( \Omega_{12} \)), and then project this spherical patch on the base plane containing \( dA_1 \), as sketched in Fig. 5a (the conical projection on the sphere is \( A_s \), and its normal projection on the base plane is \( A_p \)). The view factor \( F_{12} \) is therefore the fraction of the circle occupied by \( A_p \), i.e. \( F_{12}=A_p/(\pi r^2) \). This was originally an experimental method: if an opaque reflective hemisphere is put on top of \( dA_1 \), and a photography is taken from the zenith, the measure of the reflected image of \( A_2 \) (\( A_p \)), divided by the area of the circle, yields the view factor (in practice, the radius of the sphere was taken as unity, so the unit-sphere naming).

The same reasoning can be followed changing the intermediate hemisphere by any other convex surface covering the \( 2\pi \) steradians over \( dA_1 \), e.g. the hemicube method uses an intermediate hexahedron or cube centred in \( dA_1 \), what can simplify computations, since the discretization of the planar faces in small patches is simpler, and the view factors of every elementary patch in a rectangular grid on the top and
Radiative view factors

lateral faces of the hemicube can be precomputed; Fig. 5b shows the hemicube geometry in comparison with the unit-sphere geometry.

Fig. 5. a) The unit-sphere method, and b) its comparison with the hemicube method (Howell et al.).

WITH SPHERES

Patch to a sphere

**Frontal**

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a small planar surface facing a sphere of radius $R$, at a distance $H$ from centres, with $h = H/R$.</td>
<td>$F_{12} = \frac{1}{h^2}$</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $h=2$, $F_{12}=1/4$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Level**

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a small planar plate (one face or both) level to a sphere of radius $R$, at a distance $H$ from centres, with $h = H/R$.</td>
<td>$F_{12} = \frac{1}{\pi} \left( \arctan \frac{1 - x}{h^2} \right)$</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>with $x = \sqrt{h^2 - 1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(F_{12}</td>
<td>_{h=1} \rightarrow \frac{1}{2} - \frac{2}{\pi} \sqrt{h-1})$</td>
<td></td>
</tr>
<tr>
<td>(e.g. for $h=2$, $F_{12}=0.029$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Tilted

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
</table>
| From a small planar surface tilted to a sphere of radius $R$, at a distance $H$ from centres, with $h=H/R$; the tilting angle $\beta$ is between the normal and the line of centres. | For the facing surface:  
- if $|\beta| \leq \arccos(1/h)$ (i.e. plane not cutting the sphere),  
  $$F_{12} = \frac{\cos \beta}{h^2}$$  
- if $|\beta| > \arccos(1/h)$ (i.e. plane cutting the sphere),  
  $$F_{12} = -\frac{1}{\pi h^2} \left( \cos \beta \arccos \left( \frac{y - x \sin \beta \sqrt{1 - y^2}}{x} \right) + \frac{1}{\pi} \arctan \left( \frac{\sin \beta \sqrt{1 - y^2}}{x} \right) \right)$$  
  with $x \equiv \sqrt{h^2 - 1}$, $y \equiv -x \cot \beta$ | ![Plot](image) |
| For the whole plate (the two surfaces):  
- if $|\beta| \leq \arccos(1/h)$ (i.e. plane not cutting the sphere, hence $F_{12}=0$ for the back side),  
  $$F_{12} = \frac{\cos \beta}{2h^2}$$  
- if $|\beta| > \arccos(1/h)$ (i.e. plane cutting the sphere), $F_{12}$ can be obtained as the semisum of values for the facing surface and for the opposite surface, the latter obtained as the complement angle (i.e. with $\beta \to \pi - \beta$).  
  $$(\text{e.g. for } h=1.5 \text{ and } \beta=\pi/3 \text{ (60°)}: \ F_{12}=0.226 \text{ for the facing side, } F_{12}=0.004 \text{ for the opposite side, and } F_{12}=0.115 \text{ for the two-side plate})$$ | ![Plot](image) |

### Patch to a spherical cap

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
</table>
| From a small planar plate facing a spherical cap subtending a half-cone angle $\alpha$ (or any other surface subtending the same solid angle). | $F_{12} = \sin^2 \alpha$  
(e.g. for $\alpha=45^\circ$, $F_{12}=1/2$) | ![Plot](image) |
| Notice that the case ‘patch to frontal sphere’ above, can be recovered in our case with $\alpha_{\text{max}}=\arcsin(R/H)$. | | |
### Sphere to concentric external cylinder

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a sphere (1) to interior surface of a concentric cylinder (2) of radius $R$ and height $2H$, $h=H/R$. Sphere radius, $R_{sph}$, is irrelevant but must be $R_{sph} \leq H$.</td>
<td>$F_{12} = \frac{h}{\sqrt{1+h^2}}$ (e.g. for $H=R$, i.e. $h=1$, $F_{12}=0.707$)</td>
<td><img src="image" alt="Plot" /></td>
</tr>
</tbody>
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### Disc to frontal sphere

<table>
<thead>
<tr>
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<th>Plot</th>
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<tbody>
<tr>
<td>From a disc of radius $R_1$ to a frontal sphere of radius $R_2$ at a distance $H$ between centres (it must be $H&gt;R_1$), with $h=H/R_1$ and $r_2=R_2/R_1$.</td>
<td>$F_{12} = 2r_2^2 \left(1 - \frac{1}{\sqrt{1+\frac{1}{h^2}}} \right)$ (e.g. for $h=r_2=1$, $F_{12}=0.586$)</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>From a sphere of radius $R_1$ to a frontal disc of radius $R_2$ at a distance $H$ between centres (it must be $H&gt;R_1$, but does not depend on $R_1$), with $h=H/R_2$.</td>
<td>$F_{12} = \frac{1}{2} \left(1 - \frac{1}{\sqrt{1+\frac{1}{h^2}}} \right)$ (e.g. for $R_2=H$ and $R_1 \leq H$, $F_{12}=0.146$)</td>
<td><img src="image" alt="Plot" /></td>
</tr>
</tbody>
</table>

Radiative view factors
Cylinder to large sphere

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coaxial ((\beta=0)):</td>
<td>(F_{12} = \frac{1}{2} - \frac{s}{\pi h} \arcsin(s)) with (s = \sqrt{1 - \frac{1}{h^2}})</td>
</tr>
<tr>
<td>Perpendicular ((\beta=\pi/2)):</td>
<td>(F_{12} = \frac{4}{\pi^2} \int_0^1 \frac{x \text{E}(x)}{\sqrt{1-x^2}} \text{d}x) with elliptic integrals (\text{E}(x)).</td>
</tr>
<tr>
<td>Tilted cylinder:</td>
<td>(F_{12} = \int_{\theta=0}^{\theta=\pi} \int_{\phi=0}^{\phi=\pi} \sin(\theta) \sqrt{1-z^2} \text{d}\theta \text{d}\phi) with (z = \cos(\theta) \cos(\beta) + \cos(\phi) \sin(\beta) \cos(\phi)) (e.g. for (h=1) and any (\beta), (F_{12}=1/2))</td>
</tr>
</tbody>
</table>

Cylinder to its hemispherical closing cap

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a finite cylinder (surface 1) of radius (R) and height (H), to its hemispherical closing cap (surface 2), with (r=R/H). Let surface 3 be the base, and surface 4 the virtual base of the hemisphere.</td>
<td>(F_{11} = 1 - \frac{\rho}{2}, \quad F_{12} = F_{13} = F_{14} = \frac{\rho}{4})</td>
</tr>
<tr>
<td></td>
<td>(F_{21} = \frac{\rho}{4r}, \quad F_{22} = \frac{1}{2}, \quad F_{23} = \frac{1}{2} - \frac{\rho}{4r}, \quad F_{24} = \frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td>(F_{31} = \frac{\rho}{2r}, \quad F_{32} = 1 - \frac{\rho}{2r}, \quad F_{33} = 1 - \frac{\rho}{2r}, \quad F_{34} = 1 - \frac{\rho}{2r})</td>
</tr>
<tr>
<td></td>
<td>with (\rho = \sqrt{4r^2 + 1} - 1)</td>
</tr>
<tr>
<td></td>
<td>(e.g. for (R=H), (F_{11}=0.38, \quad F_{12}=0.31, \quad F_{21}=0.31, \quad F_{22}=0.50, \quad F_{23}=0.19, \quad F_{31}=0.62, \quad F_{32}=0.38, \quad F_{33}=0.38))</td>
</tr>
</tbody>
</table>

Radiative view factors
**Sphere to sphere**

**Small to very large**

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a small sphere of radius $R_1$ to a much larger sphere of radius $R_2$ at a distance $H$ between centres (it must be $H&gt;R_2$, but does not depend on $R_1$), with $h=H/R_2$.</td>
<td>$F_{12} = \frac{1}{2} \left(1 - \sqrt{1 - \frac{1}{h^2}}\right)$</td>
<td><img src="image1" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $H=R_2$, $F_{12}=1/2$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Equal spheres**

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a sphere of radius $R$ to an equal sphere at a distance $H$ between centres (it must be $H&gt;2R$), with $h=H/R$.</td>
<td>$F_{12} = \frac{1}{2} \left(1 - \sqrt{1 - \frac{1}{h^2}}\right)$</td>
<td><img src="image2" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $H=2R$, $F_{12}=0.067$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Concentric spheres**

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between concentric spheres of radii $R_1$ and $R_2&gt;R_1$, with $r=R_1/R_2&lt;1$.</td>
<td>$F_{12}=1$ $F_{21}=r^2$ $F_{22}=1-r^2$</td>
<td><img src="image3" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $r=1/2$, $F_{12}=1$, $F_{21}=1/4$, $F_{22}=3/4$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Hemispheres**

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Radiative view factors 13
From a hemisphere of radius $R_1$ to a larger concentric hemisphere of radius $R_2 > R_1$, with $R = R_2/R_1 > 1$. Let the closing planar annulus be surface 3.

\[ F_{12} = 1 - \frac{\rho}{4}, \quad F_{13} = \frac{\rho}{4}, \quad F_{21} = \frac{1}{R^2} \left( 1 - \frac{\rho}{4} \right), \]
\[ F_{22} = \frac{1}{2} \left( 1 - \frac{\rho}{R^2} \right), \]
\[ F_{23} = \frac{1}{2} \left( 1 - \frac{1}{R^2} \right) \left( 1 - \frac{\rho}{2(R^2 - 1)} \right), \]
\[ F_{31} = \frac{\rho}{2R^2}, \quad F_{32} = 1 - \frac{\rho}{2(R^2 - 1)} \]

with
\[ \rho = \frac{1}{2} - \frac{1}{\pi} \left[ \sqrt{R^2 - 1} - \left( R^2 - 2 \right) \arcsin \left( \frac{1}{R} \right) \right] \]

Limit for $R \to \infty$: $F_{12} = 7/8 = 0.875$, $F_{13} = 1/8$

(e.g. for $R = 2$, $F_{12} = 0.93$, $F_{21} = 0.23$, $F_{13} = 0.07$, $F_{31} = 0.05$, $F_{32} = 0.95$, $F_{23} = 0.36$, $F_{22} = 0.41$)

From a sphere of radius $R_1$ to a larger concentric hemisphere of radius $R_2 > R_1$, with $R = R_2/R_1 > 1$. Let the enclosure be ‘3’.

\[ F_{12} = 1/2, \quad F_{13} = 1/2, \quad F_{21} = 1/R^2, \]
\[ F_{23} = 1 - F_{21} - F_{22}, \quad F_{22} = \frac{1}{2} \left( 1 - \frac{1 - \rho}{R^2} \right) \]

with
\[ \rho = \frac{1}{2} - \frac{1}{\pi} \left[ \sqrt{R^2 - 1} - \left( R^2 - 2 \right) \arcsin \left( \frac{1}{R} \right) \right] \]

(e.g. for $R = 2$, $F_{12} = 1/2$, $F_{21} = 1/4$, $F_{13} = 1/2$, $F_{23} = 0.34$, $F_{22} = 0.41$)

Equal frontal hemispheres

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a hemisphere of radius $R$ to an equal frontal hemisphere at a distance $H$ between centres (it must be $H &gt; 2R$), with $h = H/R$.</td>
<td>$F_{12} = 1 - \sqrt{1 - \frac{1}{h^2}}$, (e.g. for $H = 2R$, $F_{12} = 0.134$)</td>
<td><img src="image" alt="Plot" /></td>
</tr>
</tbody>
</table>

Small hemisphere frontal to large sphere

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><img src="image" alt="Plot" /></td>
</tr>
</tbody>
</table>
From a small hemisphere (one face) to a large frontal sphere of radius $R$ at a distance $H\geq R$ between centres, with $h=H/R$.

$$F_{12} = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{1}{h^2}} + \frac{1}{2h^2} \right)$$

(e.g. for $H=R$, $F_{12}=3/4$)

### Hemisphere to plane

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
</table>
| From a hemisphere of radius $R$ (surface 1) to its base circle (surface 2). | $F_{21}=1$  
$F_{12}=A_2F_{21}/A_1=1/2$  
$F_{11}=1-F_{12}=1/2$ | |

From a convex hemisphere (1) to an infinite plane (2). Let the enclosure be ‘3’.

(coincides with Small hemisphere frontal to large sphere at $H=R$)

$F_{12}=3/4=0.75$  
$F_{13}=1/4=0.25$  
$F_{21}=0$, $F_{23}=1$  

From a concave hemisphere (1) to an infinite plane (2).

$F_{12}=F_{11'}=1/2$  
$F_{11}=1/2$

### Spherical cap to base disc

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
</table>

Radiative view factors
From a disc of radius $R$ (surface 1) to a closing spherical cap of height $H$ (surface 2), with $h=H/R$. The radius of the sphere is $(R^2+H^2)/(2H)$.

$$F_{21} = \frac{A_1}{A_2}, \quad F_{12} = \frac{R^2}{R^2 + H^2} = \frac{1}{1 + h^2} = \cos^2 \frac{\theta}{2}$$

$$F_{22} = \frac{h^2}{1 + h^2} = \sin^2 \frac{\theta}{2}$$

(e.g. for $h=1$, i.e. $\theta=90^\circ$, $F_{21}=F_{22}=1/2$)

WITH CYLINDERS

Cylinder to large sphere
See results under ‘Cases with spheres’.

Cylinder to its hemispherical closing cap
See results under ‘Cases with spheres’.

Very-long cylinders

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between concentric infinite cylinders of radii $R_1$ and $R_2 &gt; R_1$, with $r=R_1/R_2 &lt; 1$.</td>
<td>$F_{12} = 1$, $F_{21} = r$, $F_{22} = 1 - r$</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $r=1/2$, $F_{12}=1$, $F_{21}=1/2$, $F_{22}=1/4$)</td>
<td><img src="image" alt="Plot" /></td>
<td></td>
</tr>
</tbody>
</table>

Concentric cylinder to hemi-cylinder

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From very-long cylinder of radius $R_1$ to concentric hemi-cylinder of radius $R_2 &gt; R_1$, with $r=R_1/R_2 &lt; 1$. Let the enclosure be ‘3’.</td>
<td>$F_{12} = 1/2$, $F_{21} = r$, $F_{13} = 1/2$, $F_{23} = 1 - F_{21} - F_{22}$, $F_{22} = 1 - \frac{2}{\pi} \left( \sqrt{1 - r^2} + r \arcsin r \right)$</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $r=1/2$, $F_{12}=1/2$, $F_{21}=1/2$, $F_{13}=1/2$, $F_{23}=0.22$, $F_{22}=0.28$)</td>
<td><img src="image" alt="Plot" /></td>
<td></td>
</tr>
</tbody>
</table>

Radiative view factors
### Concentric frontal hemi-cylinders

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From very-long hemi-cylinder of radius ( R_1 ) to concentric hemi-cylinder of radius ( R_2 \geq R_1 ), with ( r \equiv R_1 / R_2 &lt; 1 ). Let ‘3’ be the closing surface (i.e., the two planar strips).</td>
<td>[ F_{12} = 1 - \frac{\text{arccos}(r)}{\pi} + \frac{1}{\pi r} \left( \sqrt{1-r^2} + r - 1 \right) ] (limit for ( r \to 0 ): ( F_{12} = \frac{1}{2} + \frac{1}{\pi} = 0.82 )), [ F_{11} = 0, \quad F_{13} = 1 - F_{12}, \quad F_{31} = \frac{\pi r F_{13}}{2(1-r)} ], [ F_{33} = 0, \quad F_{32} = 1 - F_{31}, \quad F_{23} = \frac{2(1-r) F_{32}}{\pi} ], [ F_{21} = r F_{12}, \quad F_{22} = 1 - F_{21} - F_{23}, ] [ F_{22} = 1 - r + \frac{2}{\pi} \left( r \text{arccos}(r) - \sqrt{1-r^2} \right) ]</td>
<td><img src="image1.png" alt="Plot" /></td>
</tr>
</tbody>
</table>

(e.g., for \( r = 0.5 \): \( F_{11} = 0.64, \quad F_{12} = 0.36, \quad F_{13} = 0 \), \( F_{21} = 0.32, \quad F_{22} = 0.36, \quad F_{23} = 0.32 \), \( F_{31} = 0, \quad F_{32} = 1, \quad F_{33} = 0 \))

### Concentric opposing hemi-cylinders

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From very-long hemi-cylinder of radius ( R_1 ) to concentric hemi-cylinder of radius ( R_2 \geq R_1 ), with ( r \equiv R_1 / R_2 &lt; 1 ). Let ‘3’ be the closing surface (i.e., the two planar strips).</td>
<td>[ F_{11} = \frac{2}{\pi} = 0.64, \quad F_{12} = 1 - \frac{2}{\pi} = 0.36, \quad F_{13} = 0, ] [ F_{21} = \frac{2r}{\pi}, \quad F_{22} = 1 - \frac{2}{\pi} = 0.36, \quad F_{23} = \frac{2(1-r)}{\pi}, ] [ F_{31} = 0, \quad F_{32} = 1, \quad F_{33} = 0 ]</td>
<td><img src="image2.png" alt="Plot" /></td>
</tr>
</tbody>
</table>

(e.g., for \( r = 0.5 \): \( F_{11} = 0.64, \quad F_{12} = 0.36, \quad F_{13} = 0 \), \( F_{21} = 0.32, \quad F_{22} = 0.36, \quad F_{23} = 0.32 \), \( F_{31} = 0, \quad F_{32} = 1, \quad F_{33} = 0 \))

### Hemi-cylinder to central strip

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From very-long hemi-cylinder of radius ( R ) to symmetrically placed central strip of width ( W ), with ( w \equiv W / R &lt; 2 ). Let the enclosure be ‘3’.</td>
<td>[ F_{11} = 1 - \frac{2}{\pi} = 0.363, \quad F_{12} = \frac{w}{\pi}, ] [ F_{13} = \frac{2 - w}{\pi} ]</td>
<td><img src="image3.png" alt="Plot" /></td>
</tr>
</tbody>
</table>

(e.g., for \( w = 1 \): \( F_{12} = F_{13} = 0.318, \quad F_{11} = 0.363 \))

Radiative view factors
**Hemi-cylinder to infinite plane**

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
</table>
| From convex hemi-cylinder to frontal plane. Let the enclosure be ‘3’. | $F_{11} = 0$  
$F_{12} = \frac{1}{2} + \frac{1}{\pi} = 0.82$  
$F_{13} = \frac{1}{2} - \frac{1}{\pi} = 0.18$ | |
| Concave side: | $F_{11} = 1 - \frac{2}{\pi} = 0.36$  
$F_{12} = F_{11'} = \frac{2}{\pi} = 0.64$  
$F_{13} = 0$ | |
| Convex side: | $F_{12'} = \frac{1}{2} - \frac{1}{\pi} = 0.18$  
$F_{13} = \frac{1}{2} + \frac{1}{\pi} = 0.82$ | |

**Equal external cylinders**

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a cylinder of radius $R$ to an equal cylinder at a distance $H$ between centres (it must be $H&gt;2R$), with $h\equiv H/R$.</td>
<td>$F_{12} = \frac{\sqrt{h^2 - 4} - h + 2\arcsin \frac{2}{h}}{2\pi}$</td>
<td></td>
</tr>
<tr>
<td>(e.g. for $H=2R$, $F_{12}=1/2-1/\pi=0.18$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. See the crossing-
Equal external hemi-cylinders

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
</table>
| From a hemi-cylinder of radius \( R \) to an equal hemi-cylinder separated a distance \( W \), with \( r=R/W \) (\( w=W/R=1/r \)). Let the strip in between be (3). | \[
F_{12} = 1 - \frac{2}{\pi} + \frac{4}{\pi r} \left( 1 - \sqrt{1 + r + r \arccos \frac{r}{2 + r}} \right)
\]
(limit for \( r \to \infty \): \( F_{12} = 1 - \frac{2}{\pi} = 0.36 \)) | \[
F_{13} = \frac{1}{2\pi r} \left( 1 + r \arccos \frac{r}{1 + r} - \sqrt{1 + 2r} \right)
\]
(limit for \( r \to 0 \): \( F_{13} = \frac{1}{4} - \frac{1}{2\pi} = 0.091 \) ) | \[
(e.g. for r=1, F_{12}=0.11, F_{13}=0.05)
\]

Planar strip to cylinder

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
</table>
| From frontal surface of strip (1) of width \( W \) to a cylinder (2) of radius \( R \) at a distance \( H \) between centres, with \( v=W/(2R) \) and \( h=H/R>1 \). | \[
F_{12} = \arctan \left( \frac{v}{h} \right)
\]
| | \[
F_{21} = \frac{\arctan \left( \frac{v}{h} \right)}{\pi} = \frac{\arctan (x)}{\pi} \text{ with } x\equiv v/h=(W/2)/H
\]
(e.g. for \( W=R=H \), i.e. \( v=1/2, h=1, x=1/2, F_{12}=0.927, F_{21}=0.148 \)) | \[
F_{12} = \arctan \left( \frac{v_2}{h} \right) - \arctan \left( \frac{v_1}{h} \right)
\]
For \( W=W_2-W_1 \): | \[
F_{21} = \frac{\arctan \left( \frac{v_2}{h} \right) - \arctan \left( \frac{v_1}{h} \right)}{2\pi}
\]
For \( W=W_2+W_1 \): | \[
F_{12} = \arctan \left( x_2 \right) + \arctan \left( x_1 \right)
\]
For \( W_1=0 \) & \( W=W_2 \) (with \( x=W/H \)): | \[
F_{21} = \frac{\arctan \left( \frac{v_2}{h} \right) - \arctan \left( \frac{v_1}{h} \right)}{2\pi}
\]
| | \[
F_{12} = \arctan \left( x_2 \right) + \arctan \left( x_1 \right)
\]
For \( W_1=0 \) & \( W=W_2 \) (with \( x=W/H \)): | \[
F_{21} = \frac{\arctan \left( \frac{v_2}{h} \right) - \arctan \left( \frac{v_1}{h} \right)}{2\pi}
\]
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Radiative view factors

Wire to parallel cylinder

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a small infinite long cylinder to an infinite long parallel cylinder of radius $R$, with a distance $H$ between axes, with $h=H/R$.</td>
<td>$F_{12} = \frac{\arcsin \left( \frac{1}{h} \right)}{\pi}$</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $H=R$, $F_{12}=1/2$)</td>
<td></td>
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</tr>
</tbody>
</table>

Finite cylinders

Base to lateral surface

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From base (1) to lateral surface (2) in a cylinder of radius $R$ and height $H$, with $r=R/H$. Let (3) be the opposite base.</td>
<td>$F_{12} = \frac{\rho}{2r}$, $F_{13} = 1 - \frac{\rho}{2r}$, $F_{21} = \frac{\rho}{4}$, $F_{22} = 1 - \frac{\rho}{2}$, $F_{23} = \frac{\rho}{4}$ [with \</td>
<td></td>
</tr><tr>
<td>ho = \sqrt{4r^2 + 1} - 1]</td>
<td><img src="image" alt="Plot" /></td>
<td></td>
</tr>
<tr>
<td>(e.g. for $R=H$, $F_{12}=0.62$, $F_{21}=0.31$, $F_{13}=0.38$, $F_{22}=0.38$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Disc to coaxial cylinder

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From disc (1) of radius $R_1$ at a distance $H_1$, to internal lateral surface (2) of a coaxial cylinder of radius $R_2&gt;R_1$ and height $H_2-H_1$, with $h_1=H_1/R_1$, $h_2=H_2/H_1$, $r=R_2/R_1$.</td>
<td>$F_{12} = \frac{1}{2} \left[ (x_2-x_1) - (y_2-y_1) \right]$ [with \ x_1 = 1 + r^2 + h_1^2, \ y_1 = \sqrt{x_1^2 - 4r^2}] [x_2 = 1 + r^2 + h_2^2, \ y_2 = \sqrt{x_2^2 - 4r^2}] [e.g. for $H_1=0$ &amp; $H_2=R_2=R_1$, i.e. for $r=1$, $h_1=0$ &amp; $h_2=1$, $F_{12}=0.62$]</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>Disc of same radius (i.e. $R_2=R_1$ and $r=1$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disc at the base (i.e. $H_1=0$, with $h=h_2$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Radiative view factors
Equal finite concentric cylinders

Between finite concentric cylinders of radius \( R_1 \) and \( R_2 > R_1 \) and height \( H \), with \( h = H / R_1 \) and \( R = R_2 / R_1 \). Let the enclosure be ‘3’. For the inside of ‘1’, see previous case.

\[
F_{12} = 1 - \frac{1}{\pi} \left( \arccos \frac{f_2 - f_4}{2h} \right), \quad F_{13} = 1 - F_{12},
\]
\[
F_{22} = 1 - \frac{1}{R} + 2 \frac{\arctan \frac{2\sqrt{R^2 - 1}}{h} - f_7}{2R}, \quad F_{23} = 1 - F_{21} - F_{22}
\]
with \( f_1 = h^2 + R^2 - 1, \ f_2 = h^2 - R^2 + 1, \)
\[
f_3 = \sqrt{(A + 2)^2 - 4R^2},
\]
\[
f_4 = f_3 \arccos\frac{f_2}{Rf_1} + f_2 \arcsin\frac{1}{R} - \frac{\pi f_1}{2},
\]
\[
f_5 = \sqrt{\frac{4R^2}{h^2} + 1}, \quad f_6 = 1 - \frac{2h^2}{R^2 (h^2 + 4R^2 - 4)},
\]
\[
f_7 = f_5 \arcsin f_6 - \arcsin \left( 1 - \frac{1}{R^2} \right) + \frac{\pi}{2} (f_5 - 1)
\]
(e.g. for \( R_2 = 2R_1 \) and \( H = 2R_1 \), \( F_{12} = 0.64, \ F_{21} = 0.34, \ F_{13} = 0.33, \ F_{23} = 0.43, \ F_{22} = 0.23 \)

Outer surface of cylinder to annular disc joining the base

From external lateral surface (1) of a cylinder of radius \( R_1 \) and height \( H \), to annular disc of radius \( R_2 \), with \( r = R_1 / R_2 \), \( h = H / R_2 \).

\[
F_{12} = \frac{y}{8rh} - \frac{(z \times \arcsin(r))}{4\pi rh}
\]
\[
+ \frac{1}{2\pi} \arccos \left( \frac{x}{y} \right)
\]
with \( x = h^2 + r^2 - 1, \ y = h^2 - r^2 + 1, \)
\[
z = \sqrt{(x + 2)^2 - 4r^2} \arccos \left( \frac{xr}{y} \right)
\]
Cylindrical rod to coaxial disc at one end

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin rod (1) of height $H$, to concentric disc (2) of radius $R$ placed at one end, with $h=H/R$.</td>
<td>$F_{12} = \frac{1}{4} - \frac{1}{2\pi} \arcsin \left( \frac{h^2 - 1}{h^2 + 1} \right)$,</td>
<td><img src="image1.png" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $R=H$, i.e. $h=1$, $F_{12}=1/4$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

WITH PLATES AND DISCS

Parallel configurations

Equal square plates

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between two identical parallel square plates of side $L$ and separation $H$, with $w=W/H$.</td>
<td>$F_{12} = \frac{1}{\pi w^2} \left( \ln \frac{x^4}{1 + 2w} + 4wy \right)$ [x \equiv \sqrt{1+w^2} \text{ and} \ y \equiv x \arctan \frac{w}{x} - \arctan w]</td>
<td><img src="image2.png" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $W=H$, $F_{12}=0.1998$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Unequal coaxial square plates

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a square plate of side $W_1$ to a coaxial square plate of side $W_2$ at separation $H$, with $w_1=W_1/H$ and $w_2=W_2/H$.</td>
<td>$F_{12} = \frac{1}{\pi w_1^2} \left( \ln \frac{P}{q} + s - t \right)$, with</td>
<td><img src="image3.png" alt="Plot" /></td>
</tr>
</tbody>
</table>

Radiative view factors
Box inside concentric box

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From an external-box face:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_{11} = 0, F_{12} = x, F_{13} = y, F_{14} = x, )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_{15} = x, F_{16} = x, F_{17} = za^2, F_{18} = r, )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_{19} = 0, F_{1,10} = r, F_{1,11} = r, F_{1,12} = r )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>From an internal-box face:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_{71} = z, F_{72} = (1 - z)/4, F_{73} = 0, F_{74} = (1 - z)/4, )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_{75} = (1 - z)/4, F_{76} = (1 - z)/4, F_{77} = 0, F_{78} = 0, )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_{79} = 0, F_{7,10} = 0, F_{7,11} = 0, F_{7,12} = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>with ( z ) given by:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( z = F_{71} = (1 - a)^2 \left( \frac{\ln p}{q} + s + t \right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p = \left( \frac{2(3 - 2a + 3a^2)}{(1 - a)^2} \right)^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( q = 2 \left( \frac{18 + 12a + 18a^2}{(1 - a)^2} \right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s = u \left( 2 \arctan \frac{2}{u} - w \arctan \frac{w}{u} \right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( t = v \left( 2 \arctan \frac{2}{v} - w \arctan \frac{w}{v} \right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( u = \sqrt{8}, v = \sqrt{8(1 + a^2)} \left( \frac{1}{1 - a} \right), w = 2 \left( \frac{1 + a}{1 - a} \right) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(A generic outer-box face #1, and its corresponding face #7 in the inner box, have been chosen.)

Radiative view factors

\[
p = \left( w_1^2 + w_2^2 + 2 \right)^2
\]
\[
q = \left( x^2 + 2 \right) \left( y^2 + 2 \right)
\]
\[
x = w_2 - w_1, \quad y = w_2 + w_1
\]
\[
s = u \left( x \arctan \frac{x}{u} - y \arctan \frac{y}{u} \right)
\]
\[
t = v \left( x \arctan \frac{x}{v} - y \arctan \frac{y}{v} \right)
\]
\[
u = \sqrt{x^2 + 4}, \quad v = \sqrt{y^2 + 4}
\]

(e.g. for \( W_1 = W_2 = H, F_{12} = 0.1998 \))
Equal rectangular plates

Between parallel equal rectangular plates of size \( W_1, W_2 \) separated a distance \( H \), with \( x=W_1/H \) and \( y=W_2/H \).

\[
F_{12} = \frac{1}{\pi xy} \left[ \ln \frac{x^2 y^2}{x_i^2 + y_i^2} - 1 ight] + 2x \left[ y_i \frac{x}{y} - \arctan \frac{x}{y} \right] + 2\pi \left[ x_i \frac{y}{x} - \arctan \frac{y}{x} \right]
\]

with \( x_i = \sqrt{1+x^2} \) and \( y_i = \sqrt{1+y^2} \)

(e.g. for \( x=y=1, F_{12}=0.1998 \))

Equal discs

Between two identical coaxial discs of radius \( R \) and separation \( H \), with \( r=R/H \).

\[
F_{12} = 1 + \frac{1 - \sqrt{4r^2 + 1}}{2r^2}
\]

(e.g. for \( r=1, F_{12}=0.382 \))

Unequal discs

From a disc of radius \( R_1 \) to a coaxial parallel disc of radius \( R_2 \) at separation \( H \), with \( r_1=R_1/H \) and \( r_2=R_2/H \).

\[
F_{12} = \frac{x-y}{2} \quad \text{with} \quad x = 1 + \frac{1}{r_1^2} + \frac{r_2^2}{r_1^2} \quad \text{and} \quad y = \sqrt{x^2 - 4r_2^2/r_1^2}
\]

(e.g. for \( r_1=r_2=1, F_{12}=0.382 \))

Radiative view factors
Strip to strip

Note. See the crossing-string method, above, for these and other geometries.

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between two identical parallel strips of width $W$ and separation $H$, with $h=H/W$.</td>
<td>$F_{12} = \sqrt{1+h^2} - h$ (e.g. for $h=1$, $F_{12}=0.414$)</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>Between two unequal parallel strips of width $W_1$ and $W_2$, and separation $H$, with $w_1=W_1/H$ and $w_2=W_2/H$.</td>
<td>$F_{12} = \frac{\sqrt{(w_1+w_2)^2 + 4}}{2w_1} - \frac{\sqrt{(w_2-w_1)^2 + 4}}{2w_1}$ (e.g. for $w_1=w_2=1$, $F_{12}=0.414$)</td>
<td><img src="image" alt="Plot" /></td>
</tr>
</tbody>
</table>

Patch to infinite plate

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a finite planar plate at a distance $H$ to an infinite plane, tilted an angle $\beta$.</td>
<td>Front side: $F_{12} = \frac{1+\cos \beta}{2}$</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td></td>
<td>Back side: $F_{12} = \frac{1-\cos \beta}{2}$</td>
<td>(e.g. for $\beta=\pi/4$ ($45^\circ$), $F_{12,\text{front}}=0.854$, $F_{12,\text{back}}=0.146$)</td>
</tr>
</tbody>
</table>

Patch to disc

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
</table>

Radiative view factors
From a patch to a parallel and concentric disc of radius $R$ at distance $H$, with $h=H/R$.

$$F_{12} = \frac{1}{1 + h^2}$$

(e.g. for $h=1$, $F_{12}=0.5$)

### Perpendicular configurations

#### Cylindrical rod to coaxial disc at tone end
(See it under ‘Cylinders’.)

#### Square plate to rectangular plate

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>From a square plate of width $W$ to an adjacent rectangles at 90º, of height $H$, with $h=H/W$.</td>
<td>$F_{12} = \frac{1}{4} + \frac{1}{\pi} \left[ h \arctan \left( \frac{1}{h} \right) - h \arctan \left( \frac{1}{h} - \frac{h^2}{4} \ln h \right) \right]$</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>with $h_1 = \sqrt{1 + h^2}$ and $h_2 = \frac{h_1^4}{h^2 (2 + h^2)}$</td>
<td></td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $h\to\infty$, $F_{12}\to1/4$, for $h=1$, $F_{12}=0.20004$, for $h=1/2$, $F_{12}=0.146$)</td>
<td></td>
<td><img src="image" alt="Plot" /></td>
</tr>
</tbody>
</table>

#### Rectangular plate to equal rectangular plate

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between adjacent equal rectangles at 90º, of height $H$ and width $L$, with $h=H/L$.</td>
<td>$F_{12} = \frac{1}{\pi} \left[ 2 \arctan \left( \frac{1}{h} \right) - \sqrt{2} \arctan \left( \frac{1}{\sqrt{2} h} \right) \right]$ + $\frac{1}{4h} \ln \left( \frac{h_2}{4h} \right)$</td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>with $h_1 = 2(1 + h^2)$ and $h_2 = \left(1 - \frac{1}{h_1}\right)^{2h-1}$</td>
<td></td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. for $h=1$, $F_{12}=0.20004$)</td>
<td></td>
<td><img src="image" alt="Plot" /></td>
</tr>
</tbody>
</table>

#### Rectangular plate to unequal rectangular plate

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><img src="image" alt="Plot" /></td>
</tr>
</tbody>
</table>
From a horizontal rectangle of \(W \cdot L\) to adjacent vertical rectangle of \(H \cdot L\), with \(h = H/L\) and \(w = W/L\).

\[
F_{12} = \frac{1}{\pi w} \left[ h \arctan \left( \frac{1}{h} \right) + w \arctan \left( \frac{1}{w} \right) - \sqrt{h^2 + w^2} \arctan \left( \frac{1}{\sqrt{h^2 + w^2}} \right) + \frac{1}{4} \ln \left( ab^2 + c^2 \right) \right]
\]

with \(a = \frac{1 + h^2}{1 + h^2 + w^2}\),

\[
b = \frac{w^2 (1 + h^2 + w^2)}{(1 + w^2)(h^2 + w^2)}, \quad c = \frac{h^2 (1 + h^2 + w^2)}{(1 + h^2)(h^2 + w^2)}
\]

(e.g. for \(h = w = 1, F_{12} = 0.20004\))

From non-adjacent rectangles, the solution can be found with view-factor algebra as shown here.

\[
F_{i+1} = F_{i+2,i+2} - F_{i+2} = \frac{A_{i+2} - A_i}{A_i} F_{2,i+1} = A_{i+2} F_{2,i+1} - A_i (F_{2,i+1} - F_{2,i+1})
\]

(e.g. \(F_{12} = \frac{1}{2} \sqrt{1 + h^2} - \frac{1}{2}\))

**Strip to strip**

Note. See the crossing-string method, above, for these and other geometries.

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjacent long strips at 90°, the first (1) of width (W) and the second (2) of width (H), with (h = H/W).</td>
<td>(F_{12} = \frac{1 + h - \sqrt{1 + h^2}}{2})</td>
<td><img src="image1" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. (F_{12}</td>
<td>_{H=H} = 1 - \frac{\sqrt{2}}{2} = 0.293))</td>
<td></td>
</tr>
</tbody>
</table>

**Tilted strip configurations**

Note. See the crossing-string method, above, for these and other geometries.

**Equal adjacent strips**

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjacent equal long strips at an angle (\alpha).</td>
<td>(F_{12} = 1 - \sin \frac{\alpha}{2})</td>
<td><img src="image2" alt="Plot" /></td>
</tr>
<tr>
<td>(e.g. (F_{12}</td>
<td>_{\alpha} = 1 - \frac{\sqrt{2}}{2} = 0.293))</td>
<td></td>
</tr>
</tbody>
</table>

Radiative view factors 27
### Triangular prism

<table>
<thead>
<tr>
<th>Case</th>
<th>View factor</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between two sides, 1 and 2, of an infinite long triangular prism of sides $L_1$, $L_2$ and $L_3$, with $h=L_2/L_1$ and $\phi$ being the angle between sides 1 and 2.</td>
<td>$F_{12} = \frac{L_1 + L_2 - L_3}{2L_1} = 1 + h - \sqrt{1 + h^2 - 2h \cos \phi}$</td>
<td><img src="image" alt="Plot" /></td>
</tr>
</tbody>
</table>

(e.g. for $h=1$ and $\phi=\pi/2$, $F_{12}=0.293$)

---

**NUMERICAL COMPUTATION**

Several numerical methods may be applied to compute view factors, i.e. to perform the integration implied in (2) from the general expression (1). Perhaps the simpler to program is the random estimation (Monte Carlo method), where the integrand in (2) is evaluated at $N$ random quadruples, $(c_{i1}, c_{i2}, c_{i3}, c_{i4})$ for $i=1..N$, where a coordinates pair (e.g. $c_{i1}, c_{i2}$) refer to a point in one of the surfaces, and the other pair $(c_{i3}, c_{i4})$ to a point in the other surface. The view factor $F_{12}$ from surface $A_1$ to surface $A_2$ is approximated by:

$$F_{12} = \frac{A_2}{N} \sum_{i=1}^{N} \frac{\cos \beta_1 \cos \beta_2}{\pi r_{12}^2}$$  \quad (11)$$

where the argument in the sum is evaluated at each ray $i$ of coordinates $(c_{i1}, c_{i2}, c_{i3}, c_{i4})$.

**Example 4.** Compute the view factor from vertical rectangle of height $H=0.1$ m and depth $L=0.8$ m, towards an adjacent horizontal rectangle of $W=0.4$ m width and the same depth. Use the Monte Carlo method, and compare with the analytical result.

**Sol.** The analytical result is obtained from the compilation above for the case of ‘With plates and discs / Perpendicular configurations / Rectangular plate to unequal rectangular plate’, obtaining, for $h=H/L=0.1/0.8=0.125$ and $w=W/L=0.4/0.8=0.5$ the analytical value $F_{12}=0.4014$ (mind that we want the view factor from the vertical to the horizontal plate, and what is compiled is the opposite, so that a reciprocity relation is to be applied).

For the numerical computation, we start by setting the argument of the sum in (11) explicitly in terms of the coordinates $(c_{i1}, c_{i2}, c_{i3}, c_{i4})$ to be used; in our case, Cartesian coordinates $(x_i, y_i, z_i, y_i')$ such that $(x_i, y_i)$ define a point in surface 1, and $(z_i, y_i')$ a point in surface 2. With that choice, $\cos \beta_1 = z/r_{12}$, $\cos \beta_2 = x/r_{12}$, and $r_{12} = \sqrt{x^2 + z^2 + (y_2 - y_1)^2}$, so that:

Radiative view factors 28
\[ F_{12} = \frac{A_2}{N} \sum_{i=1}^{N} \cos \beta_i \cos \beta_2 \left| \frac{x_2}{\pi r_{12}^2} \right| = \frac{W L}{N} \sum_{i=1}^{N} x \left| \frac{z}{\pi \sqrt{x^2 + z^2 + (y - y')^2}} \right| = \frac{W L}{N} \sum_{i=1}^{N} f_i \]

where \( f_i \) is the value of the function at a random quadruple \((x_i, y_i, z_i, y_i')\). A Matlab coding may be:

\[
\begin{align*}
W &= 0.4; L = 0.8; H = 0.1; N = 1024; \quad \text{%Data, and number of rays to be used} \\
f &= @(z,y1,x,y2) (1/pi)*x.*z./((x.^2+z.^2+(y2-y1).^2).^2); \quad \text{%Defines the function} \\
for i=1:N fi(i)=f(rand*H, rand*L, rand*W, rand*L);end; \quad \text{%Computes its values} \\
F12=(W*L/N)*sum(fi) \quad \text{%View factor estimation} \\
\end{align*}
\]

Running this code three times (it takes about 0.01 s in a PC, for \( N = 1024 \)), one may obtain for \( F_{12} \) the three values 0.36, 0.42, and 0.70, but increasing \( N \) increases accuracy, as shown in Fig. E4.

![Fig. E4 Geometry for this example (with notation used), and results of the \( F_{12} \)-computation with a number \( N=2^n \) of random quadruplets (e.g. \( N=2^{10}=1024 \) for \( i_n=10 \)); three runs are plotted, with the mean in black.](image)

REFERENCES


Back to Spacecraft Thermal Control