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PRESSURE AND PIEZOMETRY (PRESSURE MEASUREMENT)

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WHAT IS PRESSURE?

Contrary to temperature (see [Thermometry](#)), pressure is not chosen as a basic magnitude in the International System, but as a derived magnitude; pressure is surface force divided by surface area, both force and area being derived from the basic magnitudes: time, length, and mass. The metrological implications are obvious: pressure measurement reduces to force measurement on a well-defined area. In its turn, force can be measured by the acceleration of a mass or by the deformation of a body. In summary, fundamental pressure metrology is a subfield of mass metrology (usually split in two subfields: mass and density, and force and pressure).

Furthermore, as forces can ultimately be measured only in isolated bodies (that is why free-body diagrams are fundamental in Mechanics), most pressure transducers measure the difference between the pressure acting on the probe active-surface and the pressure acting on the probe rear-surface, usually the surrounding ambient pressure, yielding what is termed gauge pressure (gauge means standard); if the rear side is evacuated, then the measure is termed absolute pressure, and, if the rear side is filled and sealed to a calibrated pressure, it is termed differential pressure (the term differential pressure also applies when measuring differences between two locations within the system, as in Pitot and Venturi probes). Notice that the gauge pressure in a system varies with atmospheric conditions even when the internal pressure is constant. Sometimes the term 'relative pressure' is applied to both, gauge pressure and differential pressure; see Fig. 1 for a summary.

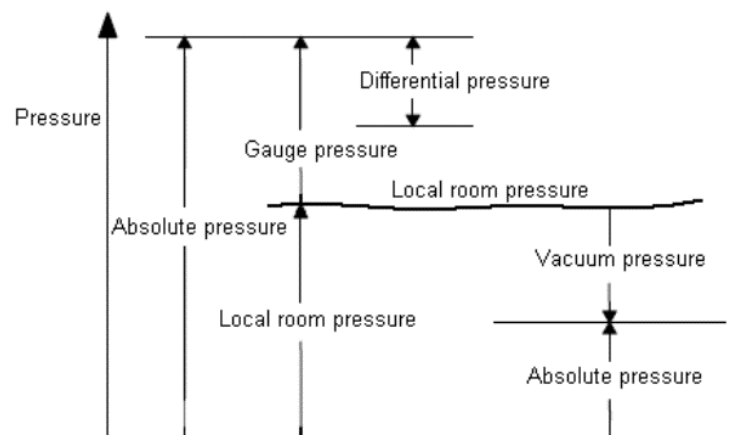


Fig. 1. Types of pressure references.

The common practice of measuring pressure differences do not imply that pressure is a relative magnitude like position or time, which depends on the reference frame or start-point chosen by the observer; pressure, like temperature (or period, length, or mass), is an absolute magnitude, as will be seen below (e.g., the length occupied by a known amount of an ideal gas inside an ideal cylinder-piston device, can be used either as a thermometer or as a piezometer). Moreover, there are no 'deltas' in the ideal gas equation $pV=mRT$; all variables there are absolute.

By the way, how can one change the pressure of a system? In a gaseous system, equation $pV=mRT$ already shows several ways: by increasing the mass of gas inside, by increasing its temperature, or by decreasing its volume. Those are static ways, as piling up more fluid or fitting a tight weight may do in a liquid. However, the common way to increase pressure in a fluid is with a rotodynamic pump, by first giving it momentum with an impeller, and then decelerating it in a diffuser. Large pressure outputs, however, usually demand volumetric-type pumps, where the momentum transfer can be performed quasi-statically.

In spite of pressure being defined in terms of force (a vector magnitude), pressure is a scalar magnitude by definition:

- For contacting solids, pressure it is just the normal force (not the total force, which may include shear) divided by the contacting area. Notice the averaging process implied, since the contact between two solids is at a wide variety of microscopic crests (varying with pressure) with air

trapped in between (if the contact were perfect, the two solids would weld together). The concept of pressure, however, does only apply to fluids and their interfaces with solids, since deformed solids are not in proper thermodynamic equilibrium and their fundamental equation, in the elastic realm, is $dU = Tds + \sum \tau_i d(c_i V_0) + \sum \mu_i dn_i$, instead of $dU = Tds - pdV + \sum \mu_i dn_i$, where τ_i are the components of the stress tensor (six, because it is symmetric), c_i are the components of the displacement tensor (six, because it is symmetric), and V_0 the un-deformed volume.

- In Thermodynamics, pressure is defined for an isotropic media at equilibrium, as $p \equiv T \partial S / \partial V|_{U, n_i}$, i.e. pressure is the sensitivity of entropy to changes in volume (or the escaping force of compression-work energy). Mechanical stability means that pressure can only be positive because, otherwise, the tendency of entropy in isolated systems to increase would mean that its volume would disappear; $dV < 0$ if $p < 0$ in $p \equiv T \partial S / \partial V|_{U, n_i}$. On the opposite, the natural tendency for systems to expand is compensated by the restrictions imposed by the surroundings. In absence of external fields, the same pressure level would be shared by any part of a system at equilibrium, but in the presence of the Earth gravity field, the hydrostatic equation applies, although, in any case, if there is an increase in pressure at any point in a confined fluid, there is an equal increase at every other point in the container at equilibrium (Pascal Principle); i.e. pressure is transmitted undiminished to all parts of the fluid and the enclosing walls (after the short transmission time proportional to the length and inversely proportional to the speed of sound in the fluid). A corollary of Pascal principle is the constant-level-tube principle. An application of it is in the hydraulic jack, in lifts, and in brakes.
- In Fluid Mechanics, for a moving fluid, pressure is defined as one third of the trace of the stress tensor changed of sign (i.e. $p = -\sum \tau_{ii} / 3 = -(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) / 3$, with σ_{ii} being dependent on flow direction, but the trace being invariant). For fluids at rest, and for flows of common fluids (air and water like) under common flow conditions, it reduces to thermodynamic-pressure, but for non-Newtonian fluids, and for the viscous flow of gases at high speeds, significant departures exist.

Pressure unit: the pascal

The symbol for pressure is lowercase p , and the unit of pressure is the pascal (Pa), which is 1 N/m^2 . The pressure at sea level on Earth is around 10^5 Pa , varying with position and time; its mean value, around $101,3 \pm 0,2 \text{ kPa}$, was established exactly as $1.01325 \cdot 10^5 \text{ Pa}$ by international agreement, based on the pressure of a 760 mm high mercury column at $0 \text{ }^\circ\text{C}$ and $g = 9.80665 \text{ m/s}^2$, what was named one standard atmosphere ($1 \text{ atm} \equiv 101.325 \text{ kPa}$). Typical weather change causes some 1 kPa pressure variations (up to 5 kPa in hurricanes). There is a trend to use 10^5 Pa as the standard value of pressure (IUPAC changed in 1982 from 1 atm to 10^5 Pa); although the substitution of 1 atm by 1 bar is insignificant in most engineering problems, notice that their difference is the typical pressure variation due to weather changes at a site, and that $1 \text{ bar} = 750 \text{ mmHg}$ (whereas $1 \text{ atm} \equiv 760 \text{ mmHg}$; the old pressure units of one millimetre of mercury column, 1 mmHg, 1 torr \equiv 1 mmHg, one metre of water column, one kilopond per square centimetre, and so on, are totally outdated). The bar ($1 \text{ bar} \equiv 10^5 \text{ Pa}$) is a non-SI units currently accepted for use with the International System, but whose use is discouraged (as well as its submultiples, as the millibar, $1 \text{ mbar} = 100 \text{ Pa}$). By the way, there is some tendency to directly substitute $1 \text{ mbar} = 1 \text{ hPa}$, but the prefix *hecto* is also discouraged

with the International System, where multiples and submultiples scaling by three orders of magnitude are favoured (at the 8th Congress of the World Meteorological Organization the hectopascal became, on 1 January 1986, the preferred unit for the measurement of pressure for meteorological purposes).

Pressure can only have positive values in a system at equilibrium, due to mechanical stability conditions. However, contrary to temperature that cannot have negative values at any instance because motion would diverge otherwise, negative pressures can be realised in nature within liquids, because the appearance of the gas phase requires the creation of new surface area, what is hindered if the liquid is so pure and the container so smooth that no heterogeneous nucleation can take place on them, and a metastable equilibrium can exist until the departure from true equilibrium is so large that homogeneous nucleation takes place. This is the most plausible explanation of how sap can go up from the roots to the top of trees higher than 10 m (the maximum suction height of an aspirating pump). The rising xylem sap is a very dilute aqueous solution; for the tallest tree, a [giant sequoia](#) 95 m tall, it would have a negative pressure of about 850 kPa at the tree crown, which is possible if heterogeneous nucleation of bubbles is avoided (much higher negative pressures have been obtained in the lab). Osmotic pumping through semi-permeable membranes, due to solute concentration differences, may have some minor contribution too, but the level of solute concentration required at the roots (about a third of the osmotic pressure of seawater), of about 10 g/L, is too large.

Standards of pressure are based on dead-weight plungers for high values, and series of expansion to sequential containers previously evacuated for medium and low values (and Boyle's law).

Very high pressures, from 1 GPa to 10 GPa, are used in geological applications, novel food processing techniques, water-jet cutting (by micro-cracking and erosion), and so on.

Pressure measurement: piezometry

Piezometry (from Gr. *pizein*, to press, and *-metron*, to measure) is the science and practice of pressure measurement, usually extended to the effects that pressure has on materials. Some common metrological characteristics, like precision and uncertainty, can be found in [Thermometry](#). The term 'piezometry' is seldom used; the term 'pressure measurement' being the usual. Some etymologies are worth recalling:

atmos-	ατμοζ	vapour
baro-	βαροζ	heaviness
mano-	μανοζ	lightness
piezo-	πιεζο	to press
vacuum	<i>vacuus</i>	emptiness

The word *barometer*, to describe the atmospheric pressure measuring instrument, is attributed to the English scientist Robert Boyle, who in a 1669 manuscript *Continuation of New Experiments* described plans for a truly portable Torricelli device. The first measure of atmospheric pressure, the most crucial event in piezometry, was performed by E. Torricelli in 1644 in the Florence Academy, with a setup similar to that shown in Fig. 2, after his master, Galileo, died in 1642. Before that, the major milestone was Hero's *Pneumatics* in the 1st century A.D.

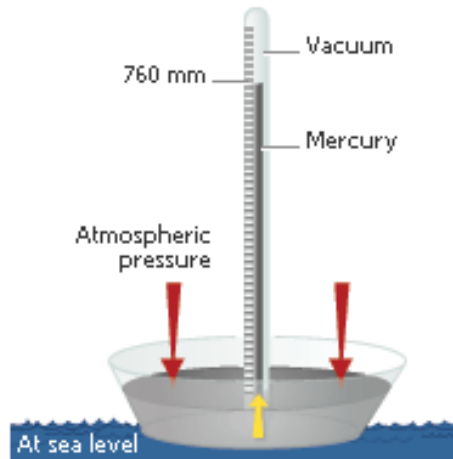


Fig. 2. Atmospheric pressure measurement (Torricelli's experiment).

And, taking about barometers, having a look at how wisely can they be made use of is a must: see ['the barometer story'](#).

Pressure measurement can be used to monitor processes (as for weather forecast) or to control them (as in a pressure cooker). The basic pressure control is the pressure switch (equivalent to the thermostat, but the word presostat is unusual in English), used to limit the pressure at a point (by switching off the pressure-building source). Two opposite pressure switches can be used to keep the pressure regulated (switching a pump on and off), although other mechanical devices (as constant-level reservoirs for liquids, or gasometers for gases) are sometimes preferred. Fluid pressure can also be controlled by diverting the fluid flow, as in relief valves and safety valves.

Pressure measurement is often used as an indirect method to evaluate other physical magnitudes, like liquid levels (and diving depth), altitude (by atmospheric pressure), fluid speed (Pitot tube), fluid flow-rates (Venturi meter), sound level (acoustics), fluid forces on an obstacle (e.g. wind loads on buildings), and so on.

VACUUM

Vacuum is empty space, i.e. a region in which gas is present at a very low pressure (no wonder why pressure cannot be properly understood without reference to vacuum, and vice versa). In the Earth atmosphere we live in, an empty container is full of ambient air; vacuum is made by further getting rid of that air as much as possible (and you cannot sweep it with another substance). There is always some matter in the highest vacuum, but it is rarefied, i.e. in a gaseous state with extremely low densities (e.g. at 400 km height (the common low-Earth orbit where the International Space Station is), pressure is in the range $10^{-7}..10^{-5}$ Pa, and density $10^{-12}..10^{-11}$ kg/m³ (but there are still some $10^{13}..10^{15}$ atoms per cubic meter!). The highest vacuum obtainable is below 10^{-8} Pa, which is produced at CERN using a cascade of vacuum technologies (primary pumps, turbomolecular pumps, sublimation pumps and ionic pumps, lasting a couple of days to evacuate 1 m³), and which could be produced at low-Earth orbit at the wake of a particle shield.

One handy way to create vacuum is with a piston-syringe device, first getting rid of air by pushing the piston against the cylinder-base while the hole is open, and then by pulling the piston away with the hole closed. Related to that is the rubber sucker, a cup-shaped device that attaches to a surface by suction, i.e. making vacuum in between and being pressed by atmospheric pressure (also used to unblock pipes). A classic pressure-and-vacuum demonstrator is the Magdeburg Hemispheres; when two small hemispheres with handles are pressed together and evacuated, they can no longer be separated by hand. Another, less obvious, way to get vacuum is to boil a low-pressure liquid in an open container and then set it off to cool down once closed (in that way, it is very easy to get less than 10 kPa by boiling some water in a glass bottle; not a trivial task with a hand-operated piston-syringe device).

Different degrees of vacuum can be defined in terms of the residual gas pressure inside, always below ambient pressure. Table 1 gives a summary of vacuum ranges, some means to get them, and some of their applications. The old pressure unit used in vacuum technology is the torr (1 torr=1 mmHg=133.32 Pa). Vacuum technology is now indispensable to various fields of scientific research as well as the medical technology, food processing, aerospace, and electronics industries.

Table 1. Types of vacuum

Types of vacuum	Pressure range	How to get it	Applications
Ambient pressure	Standard $p_{\text{atm}}=10^5$ Pa, but varies with place and time	Free available	Liquid column manometer Siphon
LV, Low vacuum or CV, Coarse vacuum	$10^3.. 10^5$ Pa	Piston pump Steam condensation Water ejector Steam ejector	Hand well pump Vacuum cleaning Refrigeration & AC Vacuum drying Vacuum distillation
MV, Medium grade vacuum	$10^{-1}.. 10^3$ Pa	Rotary oil sealed pump ^a (>10 Pa)	Freeze drying Vacuum lamps
HV, High vacuum	$10^{-7}.. 10^{-1}$ Pa	Diffusion pump ^b	Epitaxial deposition Space simulation
UHV, Ultra high vacuum	$<10^{-7}$ Pa	turbomolecular pump	High energy physics

^aThe whole mechanism of this type of pump is immersed in oil that lubricates the moving parts and also acts as the sealing agent.

^bSince these pumps only work at low pressures, the outlet of a diffusion pump must be coupled to a mechanical "backing" pump.

Vacuum generation

A vacuum system typically consists of one or more pumps which are connected to a chamber. The former produces the vacuum, the latter contains whatever apparatus requires the use of vacuum. In between the two may be different combinations of tubing, fittings (e.g. electrical feed-through for signal and power, motion feed-throughs), vacuum gauges, flow-meters and control valves.

Low grade or coarse vacuum, down to 10^3 Pa (even 10^2 Pa), may be reached using sealed reciprocating piston compressors (as are commonly found in refrigerators). Piston compressors have the disadvantage of

the dead space which exists above the piston; this dead space, plus leakage past the piston, limits the degree of vacuum that can be achieved.

Better vacuum may be obtained with a rotary, oil sealed pump, sometimes in two stages. This type of pump has a rotating off-centre cylindrical rotor that "sweeps" air through the cylindrical housing in which the rotor is located. Air is kept from passing from between the vacuum and pressure sides by means of either a set of two vanes which are arranged across the diameter of the rotor or by means of a sliding single vane mounted in the housing. The entire mechanism of this type of pump is immersed in oil. The oil lubricates the moving parts and also acts as the sealing agent.

At high vacuum, air doesn't respond very well to being squeezed and pushed around by pistons and rotors. At these pressures, gas molecules do not really flow but 'roam' into the pump. The most common type of pump for use in the high vacuum realm is the diffusion pump. This pump, invented by Irving Langmuir in 1916, utilises a jet of vapour (generated by the boiling of hydrocarbon or synthetic oil) which forces (by momentum transfer) these stray molecules into the high pressure side of the pump. Since these pumps only work at low pressures, the outlet of a diffusion pump must be coupled to a mechanical backing pump. Diffusion pumps are simple, quiet and only require simple (but sometimes tedious) maintenance. The major disadvantages are the back-streaming of oil toward the vacuum chamber (which may be minimised with baffles and/or cold traps) and the catastrophic results from accidentally opening the system to atmospheric pressure: the oil trap breaks down and goes everywhere!. Mercury was the original pumping fluid because it does not break-down and tolerates higher fore-pressures; however, mercury is toxic and has a much higher vapour pressure than diffusion-pump oils, and liquid nitrogen cold traps are mandatory to prevent contamination by back-streaming. Most of today's pumps have three stages, with inlet sizes ranging from 50 mm to 100 mm, with a pumping speed related to the inlet area of the pump (e.g. a typical speed of about $0.1 \text{ m}^3/\text{s}$ for a 50 mm inlet pump).

A variety of other styles of high-vacuum pumps have been developed, such as the turbomolecular pump (which is built roughly like a turbine), and the gas capture pumps, which can either entrap gas ions (ion pumps), freeze the gas (cryo-pumps), or bury the gas under a constantly deposited metal film (sublimation pumps).

Very high vacuum is needed in high energy particles research and in some demanding opto- and micro-electronics industries. Moderate vacuum is first made with a common vacuum pump, down to 10^{-1} Pa , from which turbo-molecular pumps can reach down to 10^{-7} Pa within several hours; finally, ionic pumps are used for several days to get to the present limit, below 10^{-8} Pa . A different approach to get such a high vacuum is by condensing the gas inside; in some outer-space simulators, the test-room, initially filled with dry nitrogen, is cooled down to 20 K with liquid hydrogen at the walls, solidifying most of the nitrogen inside (the vapour pressure of solid nitrogen at 20 K is $1.3 \cdot 10^{-8} \text{ Pa}$).

Measuring vacuum is a difficult task, typical pressure sensors being only used for coarse vacuum. For high vacuum, pressure gauges work through somewhat indirect means e.g. measuring the thermal conductivity

of the gas, or the electrical properties of the gas when ionised. The former is typically used at higher pressures ($10^{-1}..10^2$ Pa), the latter in lower ranges. Such gauges are sensitive to the type of gas in the system, requiring that corrections be made.

The flow of gases at very low pressures is very differently from that of gases at normal pressures. As a reduction in pressure occurs in a vacuum system, the gas in the system will pass through several flow regimes. At higher pressures the gas is in viscous flow where the gas behaves much like a liquid. Viscous flow includes turbulent flow, where the flow is irregular, and laminar, where the flow is regular with no eddies. Moving deeper into the vacuum environment, Knudsen or transition flow occurs when the mean free path is greater than about one-hundredth of the diameter of the tubing. Full molecular flow, where molecules behave independently, begins when the mean free path exceeds the tubing diameter. Which flow regime the gas is in is dependent upon several factors including tube diameter and pumping speed.

HYDROSTATIC PRESSURE

In the presence of a uniform force field, applicable to gravity on the Earth surface, pressure decreases with height within a fluid, in accordance with the reduction of the weight of the mass above, namely:

$$\frac{dp}{dz} = -\rho g \quad \rightarrow \quad \left\{ \begin{array}{l} \xrightarrow{PLM} \quad p = p_0 - \rho g (z - z_0) \\ \xrightarrow{PGM} \quad \frac{dp}{dz} = -\frac{p}{RT} g \end{array} \right. \quad (1)$$

i.e., in liquids, pressure increases downwards linearly (with the perfect liquid model), at a rate of 100 kPa every 10 m of water column approximately, while in gases pressure variation are so small (e.g. around 1 kPa every 100 m) that it can be neglected in all engineering problems (in meteorology, a function $T(z)$ must be prescribed to solve (1) for $p(z)$, the two most used models being the thermodynamic equilibrium $T(z)=\text{constant}$, and the average actual rate, named air standard rate, $T(z)=a+bz$ for the troposphere, with $b=-0.0065$ K/m, i.e. a 6.5 °C descent every kilometre.

Atmospheric pressure in meteorology

Atmospheric pressure was first measured by Torricelli in 1644, and it is around 100 kPa at sealevel (from 99.5 kPa to 102.5 kPa, depending on location and time), with an internationally-agreed standard value of exactly $1.01325 \cdot 10^5$ Pa corresponding to a 760 mm column of mercury at 0 °C. A typical variation in atmospheric pressure at a given site, due to weather changes, is around 1 kPa, whereas a hurricane can provoke a 5 kPa depression. Notice, by the way, that atmospheric pressure acts on the whole frontier of a body, including the area where the body contacts its supports, because the contact is not perfect and there is air in between (the two solids would weld if the contact were perfect down to molecular level, and they could only be separated by cutting).

Atmospheric pressure varies much more vertically than horizontally, but the latter is what governs winds and the associated weather transport from one place to another, no wonder why a horizontal map of isobars is the best clue to visualise that. However, weather stations are on ground at different altitudes, and station-Pressure and piezometry (pressure measurement)

measured values are corrected to sea-level values, to plot surface-level isobars; as said before, adding 1 mbar for every 10 m of elevation is a good approximation for not-too-high stations (e.g. to the average sea-level pressure of 101 kPa would correspond an average station-level pressure in Madrid, at 660 m altitude, of $101 - 6.6 = 94.4$ kPa, in good agreement with actual data: [Time series of atmospheric pressure variations in Madrid can be found aside](#)).

Remember that, because of Coriolis forces, wind blows without crossing the isobars aloft (clockwise around high pressure points in the Northern Hemisphere), but wind blows across the isobars near the surface (spiralling out of surface high pressure centres), because of boundary layer friction over the ground. Notice that, in Meteorology, isobars are shown in surface-charts, but isohypses (contours of constant height) are shown in height-charts, not only because it is easier to keep a sounding balloon at a constant pressure, but because wind modelling is easier (the equations of motion are density-independent). Usually, height contours at the 50 kPa pressure surface are plotted (e.g. 5300 m, 5500 m, 5700 m); on a pressure surface, the highest the warmer the air. The gradient of temperature from cold/low contour areas to warm/high contour areas tends to be abrupt in mid-latitudes, due to zonal convergence of airstreams, with associated winds growing stronger aloft and given way to the jet stream, i.e. a narrow region of westerly winds (moving east at some 200 km/h) at the tropopause, having an important effect on the development of air-fronts.

Liquid level measurement

Liquid level, assuming it is well-defined (i.e. without bubbling or foaming problems), can be detected optically (visually, by reflection, by refraction, by absorption, by radar), mechanically (e.g. by weighting or with a float), electrically (e.g. testing the capacitance between two coaxial cylindrical electrodes), acoustically (sonically, ultrasonically), piezometrically (differential pressure in a liquid or in a bubbling gas), X-ray or γ -ray absorption, or simply by hand, with a dipstick (or one's finger). Liquid level measurement with a piezometer is based on the hydrostatic equation, above. For instance, in a washing machine, the height of the water in the tub is measured indirectly by measuring the pressure at the bottom of the tub. The transducer can sense directly the liquid at the reference depth, or indirectly sense the required pressure to inject some inert gas at the reference depth. Notice however that the density of the test liquid must be well-known or measured for accurate level measurement. Piezometric liquid-level measurement is perhaps the second most common application of piezometry in liquids, the first being flow-rate measurement.

Archimedes' principle. Buoyancy

Hydrostatic pressure has several important effects, like the fact that liquid free surfaces are horizontal at rest, or that one cannot aspirate liquids from a deep well with a pump on the ground (you have to install the pump less than 10 m above the free surface of water, or better submerged), or the siphon effect (explained below). A most important effect, nevertheless, is the appearance of buoyancy forces on bodies within fluids (Archimedes' Principle), or in different-density fluid masses (natural convection).

Archimedes' Principle states that any object completely or partially submerged in a fluid experiences a buoyant force, due to higher pressures on the bottom than on the top, equal in magnitude to the weight of

the fluid displaced by the object (with the same direction, same centre of action and opposite sense that the weight of the fluid displaced). Besides submarines, boats and balloons, Archimedes' principle is made use of in the experimental determination of density with marked floating bodies (hydrometers or densitometers).

Weighting objects in air and water

Weight can be a very confusing concept if it is loosely understood. If one follows its definition from the 3rd CGPM held in 1901:

Weight is the force experienced by a mass at a point in Earth-surface when measured under vacuum. Conversely, weight is the force that, if applied to an isolated body, would impose on it an acceleration similar to its free fall under vacuum. By extension, it may be applied to weight over different heavenly bodies.

With this definition, a given mass (e.g. the standard kilogramme in Paris) would weight differently at different latitudes, longitudes, altitudes and times, due to geodesic and centripetal variations, and a kilogramme of cotton would seem to weight more than a kilogramme of cotton if weighted in air (similarly, if a mass of gold weights the same than a mass of silver in the atmosphere, the former will be heavier if weighted under water). The answer to the old quiz of "Is a kilogramme of metal heavier than a kilogramme of cotton?" have to be "No, if performed according to international standards; yes, if performed as usual, in air, although the difference is below 1%". The standard weight of a body is the product of its mass and the standard gravitational acceleration, $g_0 \equiv 9.806\ 65\ \text{m/s}^2$.

One may call 'apparent weight' to the force experienced by a mass on Earth when measured within an ambient fluid like air or water, but it is better to avoid confusion and just keep to the standard weight, $\vec{F}_{\text{weight}} \equiv m\vec{g}_0$.

Notice that space weightlessness (e.g. astronauts floating around) is not the lack of weight but the balancing of weight by acceleration, as in free fall (gravitational attraction at the usual 400 km of space stations is only 11% less than at sea level on ground).

Siphons

A siphon is a pipe that rises above some liquid level and discharges to a lower level (Fig. 3). To start the siphon, it must first be filled with the liquid before it is placed into position. The elevation over which a siphon will lift a liquid is limited by the atmospheric pressure to less than 10 m. If the siphon discharges into another liquid reservoir, the discharge depth must be at a level lower than that of the liquid at the intake. The siphon effect is also used as a gas seal in sanitary piping like in the toilet (were it not for the siphon seeping, the liquid in the hydraulic seal would not be completely renewed).

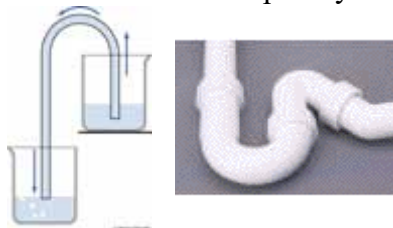


Fig. 3. Siphons: pumping siphon scheme, and picture of a sanitary siphon seal.

Inverted siphons are used to connect open channels to clear some obstacle in the path (i.e. to go under a road or a river).

PRESSURE IN THE KINETIC THEORY OF GASES

What really causes the pressure force? Could not a liquid exert zero pressure on the top of a completely-filled reservoir? (The answer is no, because at very low pressures all liquids generate vapours.) It seems important thence to analyse how a gas in a container yields pressure.

The kinetic theory of gases, assumes that gases are just a collection of point-like molecules in permanent random motion, frequently colliding with each other and with the walls of the container (without other interactions, i.e. with uniform rectilinear motion between collisions), exercising a wall pressure that is the effect of all momentum exchanges in the bouncing. It was Daniel Bernoulli, in 1738, who first proposed such a kinetic theory of gases, against the model of gases prevailing at the time: that of a highly elastic solid. In fact, it seems that sir Humphry Davy, being president of the Royal Society in the 1820's, refused to publish a first derivation of the relationship between pressure and molecular speed, on the grounds that it would imply a proportionality of temperature with internal motion, and consequently an absolute zero of temperature, contrary to established ideas at that time.

The pressure due to particle bouncing, can be computed in a crude way by noting that, if there are N particles in a cube of length L , roughly $N/6$ would be moving upwards (three axis and two senses), each one colliding with the upper wall in a time of order v/L , where v is a characteristic speed of the particles (not the mean value, which is zero at rest, but the root-mean-square value, for instance). As the momentum exchanged by each particle, of mass m , is $2mv$, the effective force is, $F=(N/6)\Delta(mv)/\Delta t=(N/6)(2mv)/(L/v)$, and the effective pressure $p=F/L^2$, equals to:

$$p = \frac{1}{3} \frac{N}{V} m v^2 \rightarrow pV = \frac{2}{3} N \frac{1}{2} m v^2 \stackrel{\frac{1}{2} m v^2 = \frac{3}{2} kT}{=} NkT = nRT \quad (2)$$

i.e., the pressure exerted by a gas is proportional to the number of particles per unit volume, N/V , and their mean kinetic energy, $E_k=(1/2)mv^2$. Further assuming that temperature is a measure of the mean kinetic energy, we come back to the ideal gas equation of state, with $k=R/N_A$ being Boltzmann's constant, $R=8.3$ J/(mol·K) the gas constant, and N_A Avogadro's number. We have not included in this model the interaction between gas particles, but this is acceptable at low pressures.

Although not directly related to pressure, a milestone in the kinetic theory of gases was set by Maxwell in 1859, and generalised by Boltzmann soon later, deducing the distribution of molecular speeds in a gas at equilibrium. The assumptions are that, at equilibrium the N particles in volume V are uniformly distributed, with their speed-direction uniformly distributed in the 4π steradians of 3-D-space (as free vectors), but without their speed-modulus uniformly distributed between 0 and ∞ (because total kinetic energy must be finite), neither constant in value (because this is not a maximum-entropy state; in fact, the distribution of

Pressure and piezometry (pressure measurement)

one-component-speed values is going to be Gaussian from $-\infty$ to $+\infty$). The probability, $p(v)$, that a particle has a speed module between v and $v+dv$, or the fraction of particles in that range, $f(v)=p(v)4\pi v^2$ (the volume of the spherical shell in velocity space), is found by solving the restricted-maximisation problem:

$$\left. \begin{aligned} \int_0^{\infty} p(v)4\pi v^2 dv &= 1 \\ \int_0^{\infty} p(v)\frac{mv^2}{2}4\pi v^2 dv &= \frac{3}{2}kT = E_k \\ \int_0^{\infty} -p(v)\ln(p(v))4\pi v^2 dv &= \max \end{aligned} \right\} \quad (3)$$

using the Lagrange multipliers method, to yield ([details aside](#)):

$$f(v) = p(v)4\pi v^2 dv = 4\pi \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} v^2 \exp\left(\frac{-mv^2}{2kT} \right) dv = \frac{2}{\pi} \sqrt{\frac{E_k}{kT}} \exp\left(\frac{-E_k}{kT} \right) d\left(\frac{E_k}{kT} \right) \quad (4)$$

Notice that the fraction of particles between v and $v+dv$, $f(v)$, increases parabolically from zero for low speeds, reaches a maximum $v_{\max} = \sqrt{2kT/m}$, and then decreases exponentially, as shown in Fig. 4, where v_{\max} has been used as dimensional parameter to have a non-dimensional plot, in spite that the root-mean-square speed $v_{\text{rms}} = \sqrt{3kT/m} = 1.22v_{\max}$ being more appropriate, or even the average value $\langle v \rangle = \sqrt{8kT/(\pi m)} = 1.13v_{\max}$.

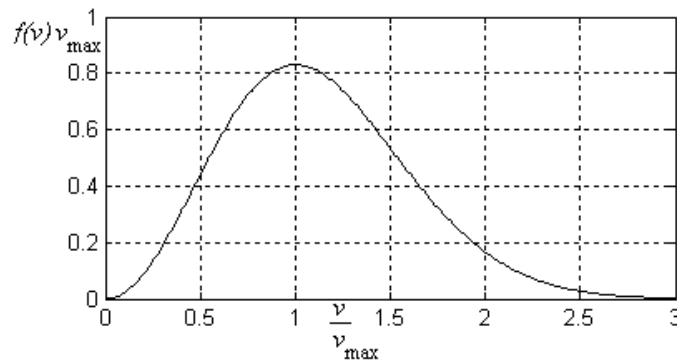


Fig. 4. Maxwell-Boltzmann distribution of molecular speeds at equilibrium.

Brownian motion, the observed constant jiggling of tiny particles within a fluid medium (e.g. pollen in water or minute fragments of ash in smoke), gives a macroscopic clue to this microscopic motion (Einstein used in 1905 Brownian motion analysis, first described by Brown in 1827, to estimate the size of atoms).

PRESSURE IN FLUID FLOW IN DUCTS

Pressure is defined as a local-equilibrium thermodynamic variable (see above), and can only be measured in such a 'static' way. Measuring pressure in ducts with fluid flow is based on the fact that cross-sectional pressure variations are negligible, and thus, a sensor flushed with the walls can be advantageously used (contrary to temperature, which can vary a lot from the wall to the centre of the duct; that is why temperature sensors are seldom in contact with walls). Both, pressure and temperature sensors within the fluid should

not distort too much the flow field, and the effects of flow speed avoided or corrected (see Dynamic pressure, below).

Bernoulli equation

Bernoulli equation (named after the Swiss physicist Daniel Bernoulli, 1700-1782), is the best known and most used equation from fluid mechanics. It is an exact first integral of the inviscid Navier-Stokes equations (Euler equations) along a streamline, relating in a simple way the fluid motion (its kinetic energy), to the thermodynamic parameters temperature, pressure, and density. Furthermore, if the flow is irrotational, the constant is the same for all streamlines, and Bernoulli's equation applies to the whole field.

$$\begin{aligned}
 \frac{D\vec{v}(s,t)}{Dt} &= \frac{\partial\vec{v}}{\partial t} + \frac{1}{2}\nabla_s(|\vec{v}|^2) = -\frac{1}{\rho}\nabla_s(p + \rho gz) \\
 \longrightarrow \int_{s=0}^{s=L} \frac{\partial\vec{v}}{\partial t} d\vec{s} &= \int_{s=0}^{s=L} \frac{\partial p}{\rho \partial s} d\vec{s} + \left[\frac{v^2}{2} + gz \right]_{s=0}^{s=L} \\
 \xrightarrow{\text{steady incompressible}} 0 &= \left[\frac{p}{\rho} + \frac{v^2}{2} + gz \right]_{s=0}^{s=L}
 \end{aligned} \tag{5}$$

Bernoulli equation can also be interpreted as a mechanical energy balance in a control volume, to get a [generalised Bernoulli's equation at steady state \(details aside\)](#):

$$w = \int \frac{dp}{\rho} + \Delta e_m + e_{mdf} \tag{6}$$

having added a possible mechanical energy exchange with the surroundings, w (a work per unit mass flow-rate), and a dissipative energy term due to mechanical degradation by friction, e_{mdf} . Equation (6) can be read as follows: when work is put into a fluid system through an impermeable wall (i.e. shaft work, w), it may be used to rise the pressure (first term; the device may be a pump or compressor), to rise the mechanical energy, $\Delta e_m = v^2/2 + gz$ (second term; the speed or the head, may be risen), and to be dissipated and incorporated to the internal energy (the last term, e_{mdf}).

The most used form of Bernoulli's equation simply reads $p + \rho gz + \rho v^2/2 = \text{constant}$ along a streamline in steady inviscid flow of an incompressible fluid in a gravity field, i.e. that total pressure (the sum of 'static pressure', 'head pressure' and 'dynamic pressure'), $p_t = p + \rho gz + \rho v^2/2$, is conserved. This is Bernoulli's equation in pressure form, as used in Aerodynamics; in Mechanical Engineering it is usually written in energy terms (dividing by ρ), and in Civil Engineering in height terms, named heads (dividing by ρg). The two basic chores in pipe-flow calculations are finding the total-pressure losses along the circuit, and matching them with the appropriate pumping. Notice that, in a piping portion without external work input, total pressure can only decrease, whereas 'static pressure' can increase at the expense of the other terms, invalidating the often-stated rule that fluids move from high pressure to low pressure.

Dynamic pressure and total pressure

The term 'dynamic pressure' here refers to the pressure-rise a moving fluid would experience 'if it were isentropically decelerated to rest'. For low-compressible motions dynamic pressure is $(1/2)\rho v^2$, where ρ is fluid density and v local fluid speed. Notice the 'isentropic' requirement in the definition; if the fluid decelerates by friction, dynamic pressure disappears by energy dissipation. The classical dynamic-pressure measuring device is the Pitot probe, and applications of dynamic pressure range from the Venturi meter to aerodynamic lift.

Notice, however, that sometimes 'dynamic pressure' refers to high-frequency periodic or transient pressure, i.e. when pressure varies significantly in short periods of time, what here would be termed dynamic measurement of pressure (instead of dynamic pressure measurement).

Pitot tube

The Pitot probe (named after the French scientist Henri Pitot, 1695-1771), Fig. 5, is a device where a small part of the fluid flow is made stagnant in a central tube, and the increase in pressure over the flowing fluid measured. Difference between stagnant pressures (also called total pressure, that exerted at the centre in a plane normal to the flow), and the undisturbed-fluid pressure (also called static pressure, that exerted on a plane parallel to the flow), yields the dynamic pressure (and from that the local flow speed; for little compressible flows $v = \sqrt{2\Delta p / \rho}$). If speed varies appreciably within the flow field, several Pitot probes, or a travelling one, can be used. A Kiel probe is a podded Pitot probe, i.e., a Pitot probe with a concentric shroud around the tip to straighten the flow locally and make it less sensitive to changes in yaw angle (i.e. to the alignment of probe with the flow direction).

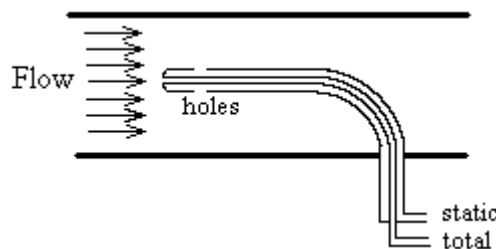


Fig. 5. Pitot probe.

Venturi effect

The Venturi effect (named after the Italian physicist G. B. Venturi, 1746-1822) is the acceleration of a confined fluid stream when the passage area decreases (not a trivial effect; one might expect the flow to decelerate when approaching a restriction, until one considers that steady state is assumed).

A Venturi nozzle, or simply a venturi, is a converging (or converging-diverging) tube used to create a depression at the neck that can be used to measure the flow-rate (Fig. 6), or to create partial vacuum for a variety of applications: liquid spraying, fluid pumping (ejectors), and so on. The Venturi effect can be measured by the simplest form of Bernoulli's equation: $p + \rho v^2/2 = \text{constant}$.

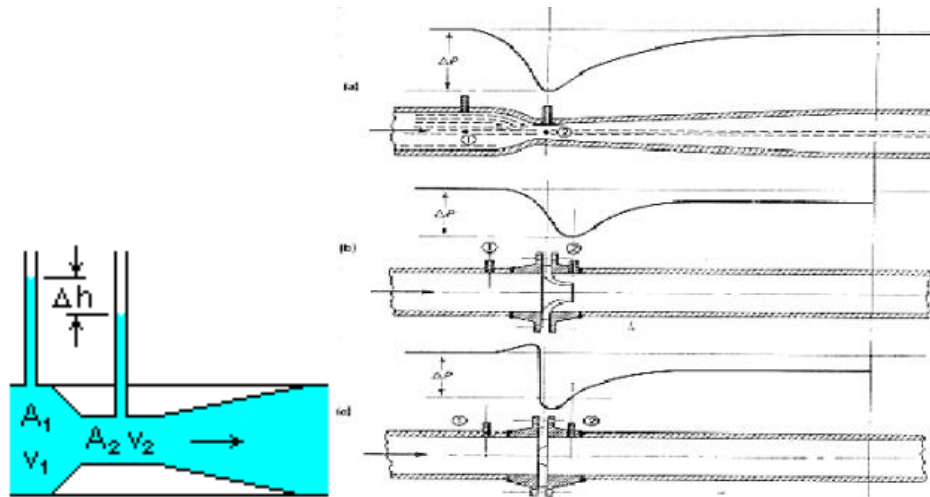


Fig. 6. Sketch of the Venturi effect, and the three types of related flowmeters: a) venturi, b) nozzle, and c) orifice.

Cavitation

Cavitation (Lat. *cavus-cavitis*, hollow) is the rapid formation and collapse of gas pockets by local depressurization in a flowing liquid until vaporisation occurs. Pure water at $T_0=15\text{ }^\circ\text{C}$ and $p_0=100\text{ kPa}$ would have to accelerate from rest to 14 m/s to cavitate, since, according to Bernoulli equation, pressure $p=p_0-\rho v^2/2$ must go down to $p_v(15\text{ }^\circ\text{C})=1.7\text{ kPa}$. However, dissolved gases in water causes cavitation to start at not-so-low pressure values, with cavities now being composed of humid air instead of pure steam.

Cavitation usually takes place in high-speed propellers at low depths (starting at the blade tip), at hydrofoil extradors, at pump intakes, at valve outlets, etc. Cavitation can be generated by an acoustic pressure field, focusing ultrasonic waves into a region in a liquid mass. Cavitation produces vibrations, noise, loss of efficiency and surface degradation by erosion. The noise in cavitation comes from the abrupt collapse of the cavity.

Cavitation can be avoided by increasing the pressure or decreasing the temperature (that is why pumps in heating systems are usually located at the cold return line). In some rare applications cavitation is wanted, as when designing for supercavitating crafts, in which a large gaseous region is created behind a sharp conical object forced to travel at very high speed; it happens that the hydrodynamic resistance of this water-separating body is much less than that of a solid shell with the shape of the whole bubble (a torpedo has been designed following that approach).

Exit pressure. Jets

Not very much related to dynamic pressure, but very important to keep in mind, is the fact that when a pipe discharges to the atmosphere (or any other large plenum), it forms a directional flow (a jet, contrary to a suction sink), and the fluid pressure at the exit mouth is precisely the plenum pressure, in the case of subsonic discharge. The reason is that there cannot be a pressure drop from the quiescent plenum fluid to the inside of the emerging jet. The discharge temperature, on the contrary, is dictated by fluid conditions upstream. Different fluid pressures upstream will force different exit velocities, but with the same (static) pressure (of course, the dynamic pressure changes).

This means, for instance, that pressure within the water jet freely issuing from a tap is always atmospheric pressure, independent of how much one opens the tap, except if the tap is fully closed; the supply pressure for a piped fluid is precisely the value that could be measured without flow. The supply pressure can be related to the speed at the exit by $\Delta p_{\text{supply}} = \Delta p_{\text{pipe-losses}} + \rho g \Delta z + \rho v_{\text{exit}}^2 / 2$, assuming the supply speed much smaller than the exit one. For instance, neglecting upstream pipe losses, the famous Geneva water-jet of $h=140$ m height requires a supply pressure of at least $\Delta p_{\text{supply}} = v_{\text{exit}}^2 / 2 = 1.4$ MPa, to provide a mouth speed of at least $v_{\text{exit}} = \sqrt{2gh} = 53$ m/s, which through an exhaust pipe of 0.1 m in diameter, ejects some 420 kg/s of water, demanding more than $\dot{W} = \dot{m} \Delta p_{\text{supply}} / \rho = 600$ kW for the pump.

Water hammer and ram pump

Most applications of Bernoulli's equation above refer to steady flow. Now we deal with the most important unsteady phenomena in duct flow: the sudden stop of a flow of liquid by closing a valve downstream, which give rise to large pressure fluctuations causing vibration, noise, and eventually pipe-wall failure.

When a pressure disturbance takes place in a fluid, it propagates in all directions with the speed of sound, c (about 1500 m/s in water). A sudden stop of a liquid flowing in a pipe gives rise to a compression of the liquid near the closure by the oncoming liquid. The simplest model may be to equate the kinetic energy of the liquid column, $E_k = \rho A L v^2 / 2$ (AL being the volume of liquid, at speed v), with the work required to compress the mass of that liquid column, $W = -\int p dV$ from an initial pressure value, p , to a final pressure value, $p + \Delta p$, i.e. $W = \Delta p \Delta V / 2$, where $\Delta V = \kappa V \Delta p$ from the [definition of the compressibility coefficient, \$\kappa\$](#) . Equating $E_k = W$ one gets ([recalling the definition of speed of sound, \$c\$](#)):

$$\Delta p = v \sqrt{\frac{\rho}{\kappa}} = \rho v c \quad (7)$$

known as Joukowsky's equation, in honour of the great Russian applied mathematician who in 1903 deduced it while investigating the water-hammer problems encountered at the Imperial Water Works in St. Petersburg (he is most famous for his later developments on aerofoil theory). For instance, if a 1 m/s pipe flow of water is suddenly stop, an abrupt pressure jump of $\Delta p = \rho v c = 1000 \cdot 1 \cdot 1500 = 1.2$ MPa, sometimes enough to cause pipe breakage.

Notice that this powerful pressure wave propagates upstream at the speed of sound, halting fluid flow at its pass. Perhaps the phenomenon is better understood considering a horizontal pipe of length $2L$ connecting two water reservoirs with a slight water-level difference to cause a quasi-steady flow (Fig. 7). When the pressure wave reaches the reservoir (in a time $t=L/c$), the wave cannot go further, the initial pressure in the reservoir is regained, reversing the flow, and the wave reflects downstream, reaching the closed valve at time $t=2L/c$ after closure, where it is again reflected, now producing a rarefaction wave that propagates upstream, reflects at the mouth, returns, and completes a full cycle in a time $t=4L/c$ after closure, which bounces back and forth. This regular bouncing back and forth is like the blows of a hammer, and the phenomenon is called water hammer. With this non-dissipating model, pressure oscillates with a square function near the valve, always with zero fluid speed there, but further away the wave is only acting for

shorter times, and, at the mouth of the pipe, there is just a pressure spike of alternating sign (see pressure record at point A in Fig. 7); the entry speed has a square function, overfilling the pipe the first L/c seconds, throwing out the excess liquid for the next $2L/c$ seconds, and so on with a full cycle of $4L/c$. At the downstream pipe in Fig. 7, a similar cycle but with opposite signs in Δp would take place. We have obviated in this crude model the possibility of negative pressures in the rarefaction wave (cavitation would take place in practice), dissipation, and pipe elasticity.

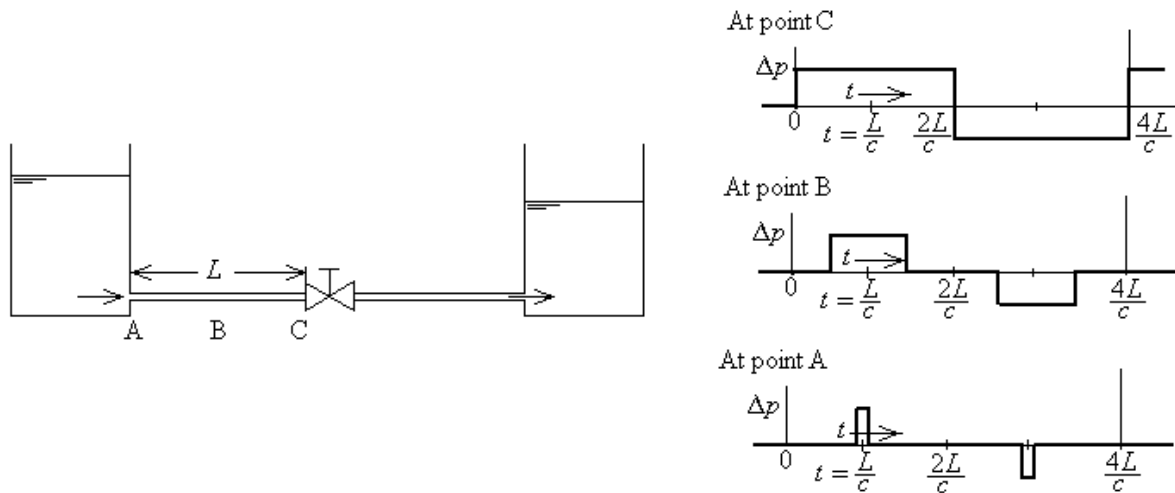


Fig. 7. Water hammer effect at different locations after valve closing at $t=0$.

Actual pipes are not rigid. When the valve is closed, the increase in pressure stretches the pipe longitudinally and a wave propagates back in the pipe more rapidly than the wave in the water, increasing the frequency of the event, but more important is the radial stretching of the pipe, which effectively reduces the compressibility coefficient from the value in the fluid, κ , to an effective global value of $\kappa' = \kappa + (5 - 4\mu)D / (4tE)$, where D is the diameter of the pipe, t its wall thickness, E Young's modulus ($E=100$ GPa for iron, 200 GPa for steel, 2.3 GPa for PVC), and μ is Poisson's ratio ($\mu=0.28$ for iron and steel, $\mu=0.5$ for PVC); for instance, in a PVC-pipe of 20 mm diameter, 2 mm thick, the speed of propagation is 500 m/s instead of the 1500 m/s in a rigid duct. Moreover, the presence of minute gas bubbles greatly decreases the propagation speed. Another complication may arise in non-straight pipes, with the appearance of lateral forces and vibrations.

Notice that, if the closure is not complete and only a sudden reduction in flow speed happens, a similar analysis works (just changing v to Δv). Notice also that 'sudden' means that the closure is performed in a time smaller than L/c .

The water-hammer effect can be profitably used as in the hydraulic ram, which is a self-powered pump based on using the energy of a large amount of water falling a small height to lift a small amount of that water to a much greater height. The first documented automatic ram pump is due to the famous Montgolfier aeronaut, who in 1796 developed it for raising water in his paper mill. Pumping operation is as follows (Fig. 8): valve V is initially open by its own weight or a spring, allowing water from the supply reservoir A to accelerate down to waste reservoir B until dynamic forces close the valve, producing a water-hammer surge that forces valve V' to open and water to rise along the delivery pipe to the high reservoir C,

decelerating until valve V opens again after a typical period of a few seconds. The objective is to raise a water flow Q a height H by using a total water flow Q_T falling a height h . By a simple exergy analysis, it has to be $Q_T h > QH$, with achieved practical efficiencies, $\eta = QH/Q_T h$, around 70%. Notice that, for a given geometry, the fraction of water flow pumped looks like Carnot efficiency: $Q/Q_T = 1 - h/H$ (exchanging heights with temperatures). In practice, a pressurized air chamber is always placed behind the delivery valve V' to make the running smoother.

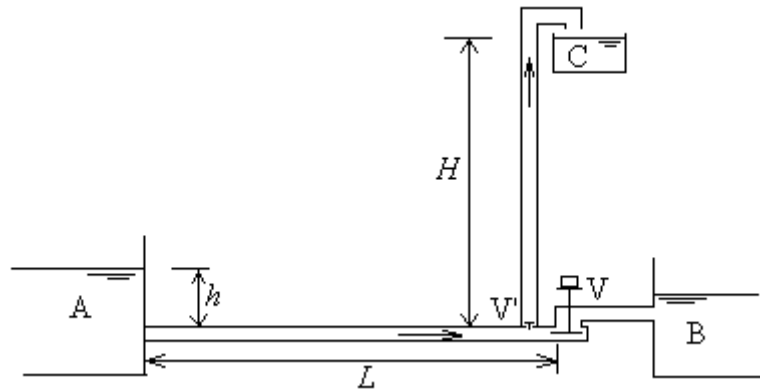


Fig. 8. Hydraulic ram pump.

PRESSURE SENSORS

Pressure measurements can be classified in terms of time, space, base reference, sensor type, fluid type, and so on. The terms static pressure and dynamic pressure do not refer here to constant and variable measurands, but are related to the effect of relative fluid speed on the measure: static pressure is the value obtained when the velocity field is undisturbed, total pressure is the value obtained when the fluid is isentropically decelerated, and dynamic pressure is the difference between both.

We said above that pressure sensors have an active face and a rear side whose pressure must be known. According to this base reference, pressure devices can be divided into different categories (Fig. 1): absolute pressure (when the reference is perfect vacuum), gauge pressure (when the reference is ambient pressure and only positive differences are measured), vacuum pressure (when the reference is ambient pressure and only negative differences are measured), and differential pressure (when the reference value is of no interest because only the difference between two values matters, as when measuring flow speed with a Pitot probe or a Venturi probe). Differential pressure devices usually work only in the positive range to avoid zero-point uncertainty, and, if one end is left open to the environment, they can either work as gauge or as vacuum pressure meters (but interchanging the connections).

According to type of sensor, pressure measurement may be based on a force transducer, or on an indirect measurement related to pressure, as in the Pirani vacuum gauge, a hot wire where its cooling rate (proportional to the thermal conductivity of the gas) is used to measure pressure (it works down to 0.1 Pa, beyond which ion gauges, based on thermoionic emission, must be used). In information process diagrams, pressure transducers are labelled PT.

The basic parameters delimiting a pressure sensor are:

- Pressure range. This refers to the operational range; most common sensors can withstand a load up to 5 times full scale without deteriorating.
- Pressure reference on the rear side of the sensor:
 - Sealed, most of the time sealed under vacuum (absolute probes), but sometimes sealed at other convenient value, e.g. 101.325 kPa (sealed probes).
 - Opened; if both sides are connected to the process it is called differential probe, but if only one is connected and the other let opened to the environment, they are called gauge or vacuum, according to the sign of the difference.
- Type of output. Piezoelectric sensors are usually passive and generate a voltage, whereas most sensors require powering at some 12 V to 50 V. Normalised outputs may be 0..5 V in tension or 4..20 mA in current (more robust to electromagnetic interference).
- Type of fluid. All probes work with inert gases, but not all of them allow liquids or dirt fluids.

Nowadays, most pressure-measuring devices are electronic, based on silicon micro-machined force transducers, offering compact, reliable, fast and cheap solutions. New developments use fibre-optics interferometry as a pressure transducer (a minute air-gap, less than 1 mm, between the fibre end and a copper diaphragm serves for the interferometric purpose). Most industrial pressure transducers look alike (Fig. 9): a round, tubular stainless steel body with a threaded pipe fitting on one end and a cable coming out of the other end.



Fig. 9. The modern industrial pressure sensor: a stainless-steel cylinder with a sensing membrane at the threaded end, and an electrical connector at the opposite end.

Recently, pressure sensor arrays have been developed to provide 'pressure images' like in thermography. The array is usually for direct contact measurement, but non-contact pressure mapping have been developed too, like the optical method based on luminescent deactivation by oxygen molecules of a photo-excited special paint that, once excited by light, shows a luminescent decaying emission that depends on oxygen concentration within the paint, proportional to pressure by Henry's law.

The classical mechanical pressure meters are the U-tube manometer and the Bourdon tube.

Liquid head transducers: the U-tube

Liquid head transducers are based on the hydrostatic equation, the paradigmatic example being the U-tube manometer. The U-tube is usually a bended glass tube in the U-shape where some manometric liquid (notably mercury or water) is poured in to fill the U-tube roughly to one half its height. The simplest U-tube is made pouring some water in a flexible hose, hold from the ends. The bore of the tube, or any cross-section detail, and even the orientation of the two branches, is irrelevant, since what matters is the difference in heights of the two menisci. Some disadvantages of U-tube manometers are:

- Small range. Only a few kPa of pressure difference can be measured using aqueous liquids, and around 100 kPa using mercury (which is discouraged). The upper range can be enlarged by using several U-tubes in series, or closing the other end of the tube. On the lower end, the smallest pressure difference is limited by meniscus stickiness to some 1 Pa even when a sloping tube is used to enlarge the sensitivity.
- Slow response. Only useful well below the 1 Hz dynamic range, depending on size and viscosity. The natural frequency in the inviscid limit is $(2g/L)^{1/2}/(2\pi)$, where g is gravitational acceleration and L the total length of the manometric column (e.g. 1.4 Hz for a $L=1$ m).
- Manometric liquid must be compatible with process fluid (e.g. should not mix or react).
- Two level measurements must be taken simultaneously (the two legs), unless a base reference is known (the zero point), or one leg is much wider than the other and its level change can be neglected.
- Requires proper orientation (cannot work horizontally) and gravity (cannot work in weightlessness).
- For very accurate work, the temperature and the relationship between density and temperature of the manometric liquid must be known.

The classical U-tube manometer (Fig. 10), will always find a place in the lab as the simplest pressure-measuring device, offering direct visualisation, and not requiring especial calibration.

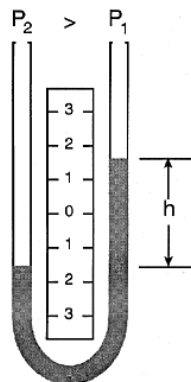


Fig. 10. U-tube manometer.

U-tube manometers can be automated, for instance by using mercury and a high-resistance wire along the interior of the U-tube (a Wheatstone bridge should be setup for high sensitivity).

A U-tube with one end closed may be used as an absolute pressure meter, but, again, similarly to Torricelli's barometer, it is impractical except with [mercury or another liquid metal](#).

Mechanical transducers: the Bourdon tube

Mechanical transducers yield a pressure signal by the elastic deformation of a capsule: a bellow, a coiled tube, a twisted tube, etc. Of course, this mechanical deformation could be further converted to an electrical signal (e.g. by driving a potentiometer rod, or a magnetic nucleus), but the probe itself does not incorporate this feature.

In the Bourdon tube, Fig. 11, fluid pressure causes the flexural deformation of a coiled tube; it was inexpensive and could be devised for widely different pressure ranges, but it is nowadays going in disuse.

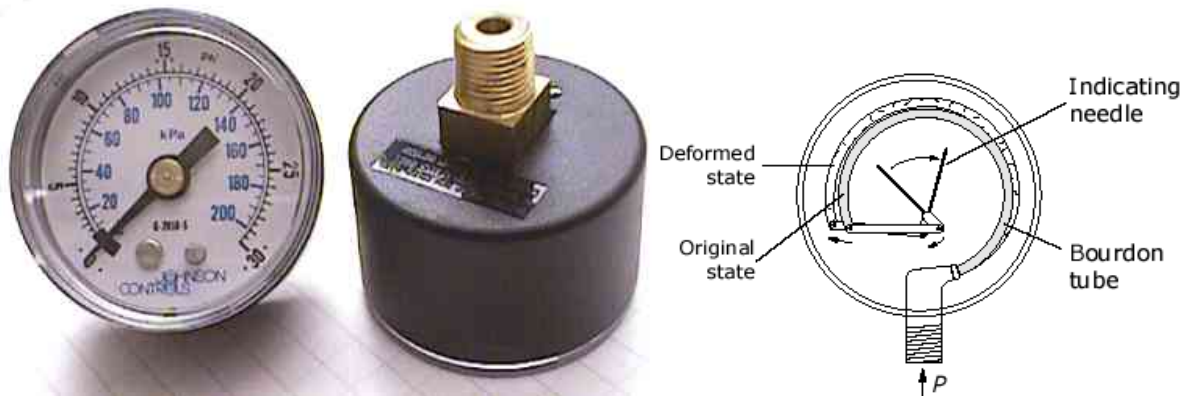


Fig. 11. Typical Bourdon tube pressure gauge (and internal details for a similar model).

Electrical transducers: membrane sensors

The common industrial pressure transducer nowadays is the solid-state membrane sensor whose minute elastic deformations are measured by an electrical change in resistance, capacitance or inductance of a suitable electrical circuit. Membranes are made of stainless steel for wide fluid compatibility and high pressure range, or made of silicon for highest accuracy (but often restricted to clean non-corrosive gases).

In a resistance-type pressure sensor, the change in electrical resistivity of a strain gauge bonded to the membrane, or better built-in within a silicon membrane, is measured with a Wheatstone bridge.

Other type of resistance transducers, as those based on a potentiometer driven by the amplified motion of the membrane or a Bourdon tube, or those based on the compression/expansion of particles whose contact resistance varies with pressure (as in the old micro-phones and ear-phones of graphite powder), are

In a capacitance-type pressure sensor, a high-frequency, high-voltage oscillator is applied to charge the sensing electrode elements, one to each side of the sensing membrane, which is typically made of stainless steel, nickel alloy or metallised silicon, and which form a condenser bridge whose unbalance is proportional to the pressure difference. Pressure ranges of capacitive sensors widespread from below 1 Pa to above 100 MPa, and have small temperature sensitivity. Capacitance-type sensors are often used as secondary standards, especially in low-differential and low-absolute pressure applications.

Electrical transducers: piezoresistive sensors

Piezoresistive pressure sensors are based on the resistivity dependence of materials under stress, similarly to strain gauges. Silicon is the most often used piezoresistive transducer. Sensors are very strong, but very sensitive to changes in temperature, so they must be temperature compensated. Transducers are available in ranges from 1 kPa to more than 100 MPa, and have the widest dynamic range (they are used to measure air-blast pressure pulses in explosions), only approached by fibre-optic sensors.

Electrical transducers: piezoelectric sensors

When pressure is applied to a quartz crystal (or natural tourmaline, or artificially polarized manmade ferroelectric ceramic materials), it is elastically deformed and electrically charged, with the charge relaxing in a few seconds. The generated voltage gives an indication of the pressure applied, although its quick relaxation makes it only useful to measure rapidly changing pressures. Piezoelectric sensors are available in ranges from 1 kPa to 100 MPa, with response time of microseconds.

Advantages of piezoelectric sensors include their rugged construction, small size, high speed, and self-generated signal. On the other hand, they are sensitive to temperature variations and require special cabling and amplification. Dynamic calibration is achieved using a dead-weight step removal, or suddenly venting a pressurised chamber, or on shock-wave tubes.

Resonant piezoelectric pressure sensors measure the variation in resonant frequency of a quartz crystal in cantilever forced to vibrate in resonance.

Non-contact pressure transducers and pressure mapping

Non-invasive pressure measurements do not sample the fluid through a hole like when using a conventional pressure meter; instead, they are based on changes in propagation of ultrasonic or electromagnetic beams, or on local electrical changes on a thin film. Non-invasive pressure methods may be applied to measures at a point, but their real advantage is when applied to bidimensional pressure mapping, that can be analogical (like in pressure sensitive paints) or already discretised (like in pressure matrix sensors). 2-D pressure measurements are most important in aerodynamics, robotics and biometrics. Even 3-D pressure maps can be constructed, as done in medical diagnostics using ultrasonic scanners.

Pressure sensitive paints (PSP) are based on the sensitivity of some luminescent dye to molecular oxygen, and the effect of pressure on the latter. The paint is essentially a luminescent dye dispersed in an oxygen permeable binder (a silicone polymer), which can be sprayed over the surface under test (previously painted white to enhance reflectivity). The dye is excited by absorbing light, usually from the blue or UV portion of the spectrum, and it then returns to its ground state by emitting light, usually in the red portion of the spectrum. There is an alternate process in which the dye can return to its ground state without emitting light by interacting with an oxygen molecule. This process is known as oxygen quenching. Thus, as the pressure of the oxygen above the PSP increases, the oxygen concentration within the binder will increase, and the intensity of the emitted radiation will decrease. Unfortunately, PSP can easily get contaminated, degrade with time of exposure to the excitation illumination because of photochemical reactions, and their output depends on temperature too.

Pressure sensor arrays consist of a matrix of pressure sensitive elements; electrical capacitance or electrical resistance sensors are commonly used. Sensors can be built using overlapping electrodes at the junction when driving strips overlap perpendicular sensing strips; by selectively measuring the capacitance or the resistance through an electrode with scanning electronics, a pressure map can be visualised.

Invasive pressure mapping has been traditionally used in aerodynamics by means of an array of fine metal or plastic tubes embedded in the model, and an array of U-tube legs.

Laboratory practice

Pressure measurement can and should be practiced when studying Fluid Mechanics, but it is also a must for a Thermal Engineering laboratory we are considering here. As a quick-check list of what we are covering in our lab demonstrations, we have:

- The U-tube manometer and manual pressure and vacuum pumps. How a U-tube can be used: pressure range, precision and accuracy, dynamic range, materials compatibility. Find its natural frequency.
- Compare the reading of several manometers (of the same and different types), of mechanical type (Bourdon tube) and of electrical type (condenser membrane). Let the student calibrate a manometer against the U-tube. Apply them to measure air under pressure and under vacuum in a bottle, and to measure water depth in a reservoir.
- Use piezometers to check Boyle's law in a gas inside a syringe ($pV=\text{const.}$ if $T=\text{const.}$).
- Use piezometers to find the ratio of thermal capacities, $\gamma \equiv c_p/c_v$, by Clément-Désormes method and by Rüchards method. Uncertainty analysis.
- Use vacuum to force boiling at low temperatures (the equipment in Fig. 12 is used).

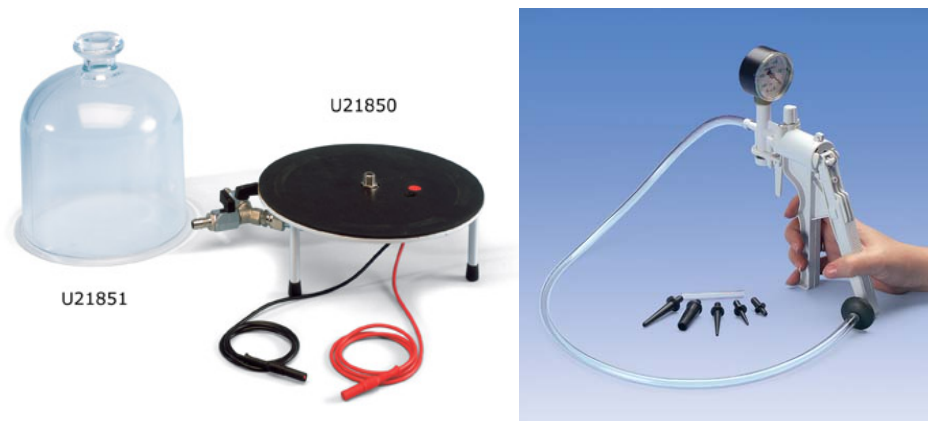


Fig. 12. A vacuum bell with its base plate, and a manual vacuum pump.

- Use piezometers to measure changing liquid level in a tall tank; try to see the effect of waves.
- Use piezometers to measure fluid speed with a Pitot tube, and flow-rate by pressure loss with a Venturi meter.
- The vapour-pressure probe thermometer. The ignition pressure tube (Fig. 13).



Fig. 13. A compression igniter.

PRESSURE COOKERS AND BOILERS

A pressure cooker is a strong hermetically sealed pot in which food may be cooked quickly under pressure at a temperature above the normal boiling point of water, i.e. a saucepan with a locking lid that keeps steam hotter than in a conventional pot. There are currently two types of pressure cookers, according to the pressure level: classic type, where $\Delta p < 0.1$ MPa (i.e. less than 100 kPa overpressure), and quick type, with $\Delta p > 0.1$ MPa.

A boiler is usually a pressure vessel or arrangement of enclosed pressure tubes in which water is heated to supply steam (to drive an engine or turbine, or to provide heat), although some other times it refers to a domestic heater burning solid fuel, gas, or oil, to provide hot water, especially for central heating. Boilers are classified by pressure level (over atmospheric pressure) as low pressure boilers ($\Delta p < 0.1$ MPa), and high pressure boilers ($\Delta p > 0.1$ MPa), the latter being the most used in industry. Pressure vessels are all routinely tested to a so called 'proof pressure' level, with some statistical samples destructively tested to 'burst pressure' level. Those tests are done with water under pressure, since doing it with a gas or vapour would be unsafe.

The first pressure vessel, with the first safety valve, is due to Denis Papin, from 1679. Gas-pressurised, vapour pressurised and hot-water pressurised vessels, may explode because they have a lot of exergy, contrary to liquid filled tanks, which just burst under pressure without explosion. Two types of safety devices are commonly used:

- Relief valves (pressure regulators); they open proportionally to excess pressure or under command, with a hissing sound.
- Safety valves (usually several, stepped); they open fully, with a loud pop sound. They can be:
 - Reversible (closes after low-pressure recovery; e.g. spring-loaded type). May be tested.
 - Irreversible (abnormal operation; permanent damage; must be replaced). Statistical test.

Boilers were used for cooking purposes since Papin invention, but soon their industrial interest switched to vapour generation to drive steam engines. The term pressure cooker first appeared in print in 1915, and it referred to large pots used to quickly boil canned food (the most important food preservation technique since Napoleon's days). The first domestic pressure cooker for preparing meals rather than canning, was presented at the New York World's Fair in 1939.

Pressure cooker safety valves

Many accidents were caused in the past by opening the pot while under pressure, releasing burning vapours, and by excessive pressure building up and blowing off the lid, injuring people. The following safety measures are implemented nowadays:

- Lid-lock. Modern pressure cookers are designed to make impossible the opening under pressure (a pressure lock must first be opened, the lid-lock, and vapour would be ejected away of the operator, if excessive; then the lid must be rotated out of the rim-lid interlocking). Air trapped when closing the pot must be vented for pressure cooking. The vent line (or vent pipe) slowly

vents air during heating, with a soft blowing sound, but would hiss when excessive pressure builds up.

- Pressure indicator. A pop-up mechanism (also in the handle) is a permanent indicator of the pressure level inside. Modern models have two pressure levels marked in the pop-up, usually for $p=150$ kPa (112 °C) and $p=200$ kPa (120 °C); remember that, after deaeration, water temperature and pressure are related through the saturated vapour curve in absolute values (not relative to the atmospheric pressure).
- Pressure regulator. This is a vapour release valve for normal operation (but works also as a safety device). In the classic-type pressure-cooker, a weight-loaded valve (petcock) of mass m seating over a vent tube of cross-section A , spinning for stability, keeps internal pressure to approximately $\Delta p=mg/A$ (operating pressure is around $\Delta p=80$ kPa, some 117 °C). Modern-type pressure cookers work as a closed system (nearly without releasing vapour, once the air vented) regulating pressure with a spring-loaded valve (connected to the pop-up indicator). Notice that, in any case, pressure regulation only works for small pressure variations, so that the heat input must be closely matched to heat losses to ambient air (plus vapour losses in the classical models); otherwise, either internal pressure would die out for not-enough heating, or pressure would build up, releasing vapour with a loud hissing and even triggering the safety valve.
- Pressure release valve for normal operation. The operator may want to manually release pressure without stopping the cooking (e.g. to add ingredients or to check for doneness), what must always be done not too fast, to avoid the violent internal bubbling to clog the vent pipe. In classic models, it was extremely dangerous to take out the petcock valve (danger of vapour burns, venting boiling liquid and vent clogging), but on modern models, the vapour release can be safely done through the pressure release mechanism (at the base of the regulator), with the escape pointing away from the operator; however, the release must be very slow if the pot contains so much bubbling stuff that liquid could reach the vent.
- Two stages of abnormal operation safety features:
 - If $\Delta p=150$ kPa (or 120 kPa in classic models), the first safety valve opens without spoiling the device, usually by means of a spring-loaded reversible valve, or by an elastic rubber-cap that ejects.
 - If $\Delta p=300$ kPa, the second safety valve safely bursts, irreversibly spoiling it. It may be an elastic rubber that yields or ruptures (at the lid or through a window in the lid-rim joint), a soft metal notch that melts, or a clamp that yields.

After the cooking time, pressure can be quickly released (and cooking stopped, what is important, e.g., to avoid fish and vegetables becoming overcooked) by cooling the pot with tap water at the sink, or pressure slowly released by ambient air-cooling. In any case, air must be allowed to enter the pot to avoid the high vacuum the cooling would cause, so, when cooling under the tap, never point the water jet over the pressure regulator valve. Besides, quick cooling often causes the skin of legumes to detach.

ACOUSTIC PRESSURE: MICROPHONES

The propagation of small periodic disturbances in the pressure and density of a fluid, in the form of longitudinal elastic waves, is known as acoustics, whereas its perception by the human ear is called sound. Acoustic waves are characterised by three basic independent parameters: frequency, amplitude, and speed.

Acoustic waves propagate at a well-defined speed, $c \equiv \sqrt{\partial p / \partial \rho|_s} = 1 / \sqrt{\rho \kappa_s}$, where κ_s is the isentropic compressibility coefficient, which is nearly constant for [liquids](#) and [solids](#) (e.g. 1500 m/s in water, 5700 m/s in stainless steel, and varies as $c = \sqrt{\gamma RT}$ for perfect [gases](#) (e.g. 340 m/s in air at 15 °C, first measured in 1627 by F. Bacon). The sound speed, c , refers to a medium at equilibrium, and thence at rest. If the speed is measured with a relative motion between sound-source, propagating-medium, and sound-detector, thence, Doppler effects appear (what can be used for anemometry, for instance).

Frequency is perhaps the key characteristic of acoustic waves (as for electromagnetic waves). Most acoustic waves are not single-frequency but are composed of a frequency band with characteristic spectra (frequency distribution).

- Below 20 Hz there is little interest for acoustics (it may be important in mechanical vibrators and seismology), basically because very large receptors are required to detect small amplitude waves (large values are detected using traditional pressure transducers). The size of the receptor must be proportional to the wavelength, $\lambda = c/f$, that for a 50 Hz wave in air means $\lambda = 340/50 = 7$ m, or $\lambda = 1500/50 = 30$ m within water.
- From 20 Hz to 20 kHz is the usual sound spectrum (where humans can hear, although sensitivity at the ends of the band greatly decreases).
- From 20 kHz to 1 MHz is the ultrasonic range, used in echo sounding, medical diagnostics, metal cleaning, and so on.
- Above 1 MHz there is little interest for acoustics because attenuation is so great that prevents propagation in practice.

The amplitude (or intensity) of acoustic waves can be measured using different parameters:

- Pressure amplitude over the undisturbed ambient pressure, $\Delta p_{\text{rms}} = \sqrt{\langle (p - p_{\text{amb}})^2 \rangle}$ (notice that peak-to-peak amplitude would be around three-times larger, being equal to $\pi \Delta p_{\text{rms}}$). Human ear works well between $\Delta p_{\text{rms}} = 20 \cdot 10^{-6}$ Pa and 20 Pa, suffering if above. The energy carried in the wave is the product of the overpressure and the particle displacement, thence proportional to the square of the perturbation amplitude, or $(\Delta p_{\text{rms}})^2$, flowing in the direction of propagation. To the human thresholds above, $\Delta p_{\text{rms}} = 20 \cdot 10^{-6}$ Pa and 20 Pa, correspond energy thresholds $w_{\text{th}} = 1 \cdot 10^{-12}$ W/m² and $w_{\text{th}} = 1$ W/m². As animal sensitivity decays with the stimuli, sound levels are always measured in a logarithmic scale (see below).
- Sound pressure level (SPL), is the non-dimensional pressure amplitude relative to a threshold internationally agreed to be $\Delta p_{\text{th}} = 20 \cdot 10^{-6}$ Pa for atmosphere acoustics, and $\Delta p_{\text{th}} = 1 \cdot 10^{-6}$ Pa for under-water acoustics, and defined by $\text{SPL} \equiv 20 \log_{10}(\Delta p_{\text{rms}} / \Delta p_{\text{th}})$. The non-dimensional unit so defined has a special name: decibel, dB, in honour of A.G. Bells (in that way, normal sounds are in the range 0..120 dB (corresponding to $\Delta p_{\text{rms}} = 20 \cdot 10^{-6}$ Pa and 20 Pa, with a discrimination

around 1 dB, i.e. the human ear needs some 12% increase or decrease in pressure amplitude to distinguish sounds: $20\log_{10}1.12=1$ dB). Sometimes sound pressure levels are corrected by a standard human ear-sensitivity spectrum normalised to a 1 kHz sound (see below).

- Sound intensity level (SIL), also known as sound power level, is the non-dimensional propagation energy relative to a standard threshold of $w_{th}=1\cdot 10^{-12}$ W/m², and defined by, $SIL=10\log_{10}(w_{rms}/w_{th})$, which coincides with sound-pressure-level definition above.. Notice that two equal-sources add up with a 3 dB increase over a single one ($10\log_{10}2=3$ dB). Sometimes, power levels are corrected by a standard human ear-sensitivity spectrum normalised to a 1 kHz sound, and, to make it clearer, the unit is named 'phon' instead of decibel (1 phon equals 1 dB at 1 kHz), see Fig. 14 and Table 2.

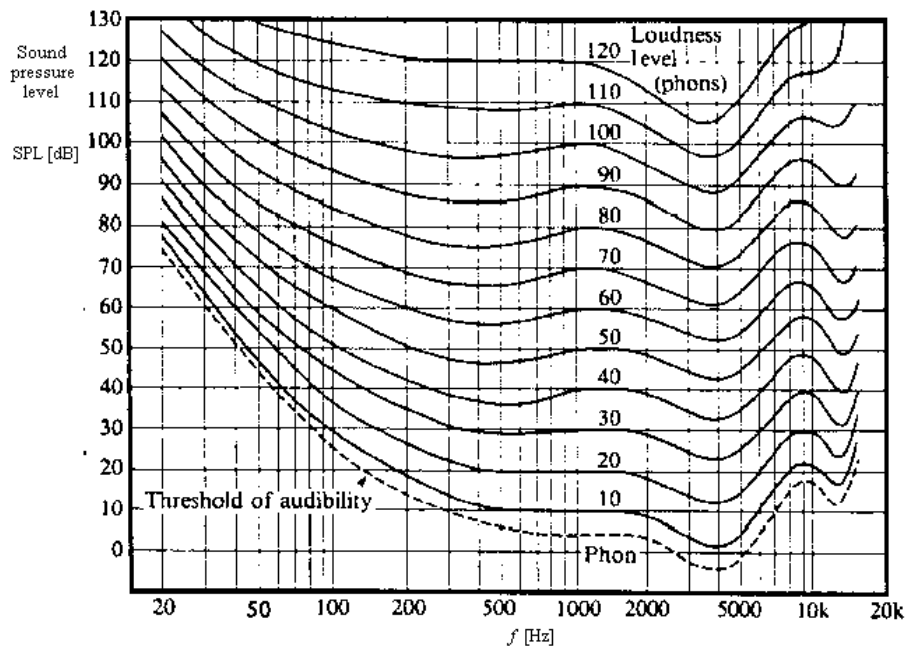


Fig. 14. Corrections of acoustic amplitude levels to sound levels, according to frequency.

Table 2. Some typical sound values.

Sounds	Sound level [dB]	Δp_{rms} [Pa]	w [W/m ²]
Imperceptible	0	$20\cdot 10^{-6}$	10^{-12}
Normal conversation	60	$20\cdot 10^{-3}$	10^{-6}
Pain threshold	120	20	1
Pneumatic hammer	130	60	3
Irreversible damage	140	200	10

Notice that the sound power in gases is very low, being of interest only as an information carrier but without any energy application (only supersonic waves have important energy effects), contrary to sound power in liquids, which can be powerful enough to break pipes. Sound attenuation with distance (i.e. the decrease in sound level) is basically due to three-dimensional spreading, because absorption coefficient for air is some $10^{-10}f^2$ dB/m (e.g. a 1 kHz planar sound-wave would have to travel some 10 km to halve its initial intensity), being due to viscosity, heat conduction, and molecular mean free path. Absorption in polar liquids is higher than in non-polar liquids, and larger than in gases. Large sound absorption takes place at interfaces. Sounds in the atmosphere are further attenuated by the vertical temperature gradient, which bends upwards the

direction of propagation creating a shadow at ground level (that is why, raising the source of sound, like for bells in church towers, increases the range).

Sound emitters are usually mechanical vibrators electrically driven: loudspeaker.

Sound receptors are high frequency pressure transducers, traditionally called microphones for sounds in air, and hydrophones for sounds in water. Perhaps the best wide-range microphone is the condenser microphone, based on the variation of electrical capacitance with separation between two charged plates, one of them being a very thin membrane to be sensitive to small pressure changes.

In human hearing, what matters is the effective sound level, L , which is the physical intensity, I , corrected by an average human sensitivity ratio, relative to 1 kHz, similarly to the change from radiometric units to photometric units.

BLOOD PRESSURE

Blood pressure in humans change with position and time, always being above atmospheric pressure, and it is usually characterised by its maximum and minimum value in the brachial artery (in the arm), at heart's height, on a seated person at rest. Pressure in other large arteries is nearly the same, and measuring at heart's height avoids hydrostatic effects. Arterial pressure measurement is a classical medical diagnostic tool since 1905; venous pressure measurement is also used in intensive care medicine, but requires invasive techniques. Blood pressure values are traditionally given in mmHg units (in fine analysis), in cmHg (in ordinary language) or in SI-units. Typical adult values are $\Delta p_{SP}=12 \text{ cmHg}=120 \text{ mmHg}=16 \text{ kPa}$ for systolic pressure (SP), and $\Delta p_{DP}=8 \text{ cmHg}=80 \text{ mmHg}=11 \text{ kPa}$ for diastolic pressure (DP); those 8-12 figures for adults, change to 5-8 for children, and 9-13 for third-age people, showing smaller variations with sex and race. Mean arterial pressure (MAP) is defined as $\Delta p_{MAP}=\Delta p_{DP}+(\Delta p_{SP}-\Delta p_{DP})/3$, and pulse pressure (PP) as $\Delta p_{PP}=\Delta p_{SP}-\Delta p_{DP}$. Blood is the most pressurised body-fluid in a person; eye pressure and bladder pressure are usually below $\Delta p=3 \text{ kPa}$, encephalic liquid below $\Delta p=1.5 \text{ kPa}$, etc.

There are three different methods in use to measure blood pressure: invasive probe (a catheter with an isodense saline solution, only used during operations), the traditional auscultatory method based on hearing sounds through a stethoscope using a sphygmomanometer cuff (that is inflated for a few seconds so as to squeeze the artery in the arm to shut, Fig. 15), and the modern oscillometric method based on pressure pulse measurement in a cuff (Fig. 16).

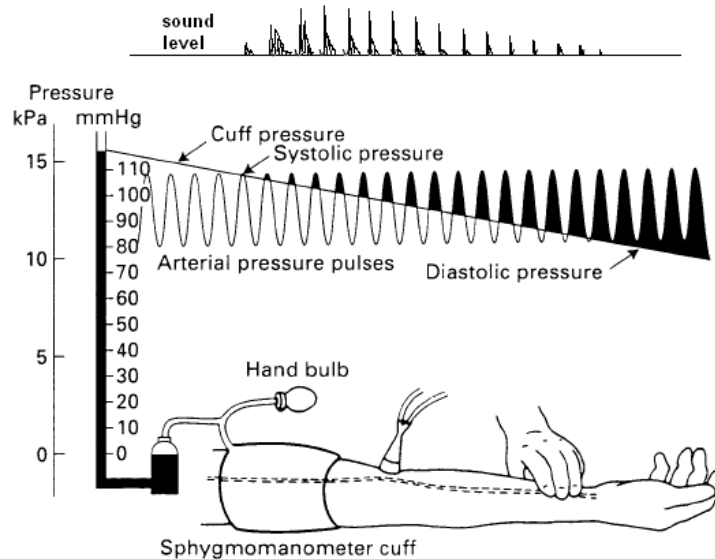


Fig. 15. Principle of auscultatory blood pressure measurement.

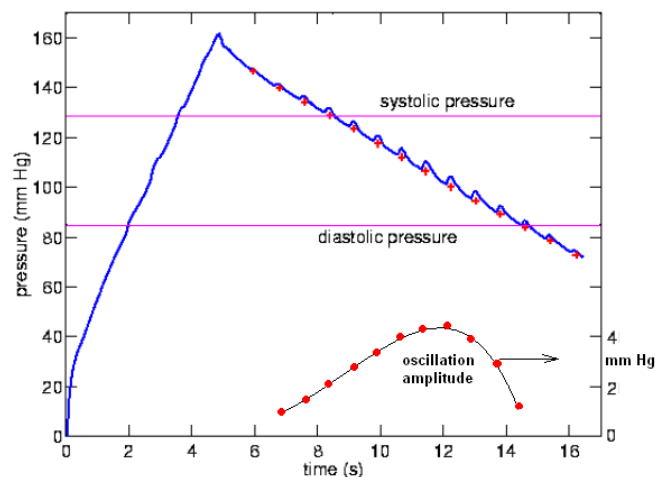


Fig. 16. Principle of oscillometric blood pressure measurement.

Hypertension is having reproducible blood pressure values of >140 mm Hg systolic and/or >90 mm Hg diastolic. Three levels of severity are considered: mild hypertension ($140..160$ mmHg and/or $90..100$ mmHg), severe ($160..180$ mmHg and/or $100..110$ mmHg), and critical (>180 mmHg and/or >110 mmHg). There are not special symptoms to hypertension. When blood pressure is elevated for an extended period of time, the inner linings of the coronary arteries become damaged, leaving them susceptible to the build-up of fatty deposits that can narrow or block the arteries and reduce blood flow to the body's organs. High blood pressure can lead to heart failure, stroke, kidney damage, and loss of vision from damage to the retina at the back of the eye.

The maximum blood pressure in mammals is found in the giraffe, with a mean arterial pressure value of $\Delta p_{MAP}=23$ kPa (28 cmHg) able to guarantee a brain pressure of $\Delta p_{brain}>8$ kPa (>6 cmHg), while the head is nearly 3 m above.

REFERENCES

<http://www.npl.co.uk/pressure/barometry.html>.

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