The unique characteristic of space propulsion is the absence of a material medium (i.e. vacuum conditions), which prevents the throwing back of environmental matter for the vehicle to accelerate forward, with the following major consequences:

1. Propulsion under vacuum requires the throwing back of on-board mass. We call 'propellants' the materials to throw, and 'rocket' the engine to force the ejection; the energy to power the rocket
engine may come from inside the vehicle (e.g. combustion) or from outside (e.g. solar power); see Propulsion fundamentals, aside.

2. Propulsion under vacuum requires the carrying of both fuel and oxidiser aboard. This is a huge penalty in space propulsion, when we recall that in the burning of most fuels, stoichiometry dictates that every kilogramme of fuel requires 15 kg of air. Yes, we do not need to transport air but only oxygen, or any other oxidiser; and we might totally forget about combustion, and use other physical process to accelerate the propellants backwards to achieve propulsion.

3. Propulsion under vacuum requires the throwing of on-board mass to decelerate and brake, since there is no air drag.

But, before we go on, we should specify what 'space' means in Space propulsion. By default, we understand 'outer space' where vacuum prevails, say beyond the 100 km altitude Kárman line on Earth until another planetary atmosphere is encountered; see Space environment. Besides this vast genuine outer space, we are also including under Space propulsion the atmospheric flight of space vehicles (e.g. launchers and descenders).

Contrary to aircraft flight, which must be continuously propelled (sailplanes considered aside), spacecraft flight is most of the time unpowered (ballistic motion dominated by gravity forces), with some periods of propulsion based on rocket engines, i.e. on high-speed ejection of some stored mass (the propellant), accelerated by internal energy sources (e.g. chemical or nuclear reactions) or by external energy sources (e.g. natural radiation from the Sun, or radiation coming from a 'power station' spacecraft, originating in a nuclear reactor or stored solar radiation, being aimed to the recipient spacecraft in a laser or microwave beam). Another distinct feature of space propulsion is that, for the time being, there is no refuelling possibilities, and the end of a space missions is often dictated by fuel run out.

We only deal here with space-related rockets (launchers and space thrusters, including sounding rockets), but rockets are not only used in spacecraft propulsion, but in some aeronautical applications (notably in missiles). Rockets for military and recreational uses (fireworks) date back to at least 13th century China. Relative motion under the outer-space vacuum must be created by ejecting own mass backwards. A rocket is the 'engine' to achieve that, although it may simply be a gas reservoir with a hole (like when an inflated balloon-toy is released open-mouth). Other means of spacecraft propulsion without mass ejection can be conceived, by making use of environmental material and electromagnetic flows in outer space (e.g. solar-sail propulsion), or the ejection of electromagnetic radiation (a flow of energy without rest-mass but with momentum), but the forces involved are too small for present applications.

Space propulsion is needed for launching from a planet surface to orbit (suborbital flights included), orbit acquisition and orbit keeping against perturbations, orbit changes (including interplanetary transfers), deorbiting (descent and landing on the surface of a planet or moon), and attitude control in all phases of space flight (launch, orbiting, and deorbit).

The first rocket theory is due to K.E. Tsiolkovsky (a Russian teacher, reader of Jules Verne's 'From Earth to the Moon', who developed it in 1880s and published it in 1903). He derived the rocket equation (presented Spacecraft propulsion
below), rediscovered in 1923 by H. Oberth. First practitioners who developed rocket technology were R.H. Goddard in 1926, W. von Braun in 1943 (he developed the first reliable rocket, the V-2, the first device to reach outer space, \( z > 100 \) km, in 1944, and the first to carry living beings there: several fruit flies were loaded in a US-launched V-2 in 1947, and recovered alive), and Sergei Korolev in 1957, who leaded the Soyuz rocket development (the most used launcher in the world, with more than 1700 flights since its introduction in 1966).

Some frequently asked questions

Q. Why there is no other solution but rockets to go to outer space?
A. We only know that way: a long-lasting ejection of mass carried aboard (rocket) to overcome Earth's attraction and gain the huge speed needed to go around (orbit). Other possible ways, as a cannon shot without further propulsion, an aircraft that would store some incoming air while ascending (for later propulsion under vacuum), climbing up a permanent rope with its centre of mass at geostationary altitude (or a space elevator), are out of reach.

Q. We all know that the first person to go to outer space was Yuri Gagarin in 1961, but ¿when did the first animal fly into space?
A. It was in 1947; a set of fruit flies in a US-launched V-2 rocket reaching 109 km altitude (i.e. over the 100 km agreed space boundary). The Laika dog was the first animal to go into orbit, on Sputnik-2, just 30 days after the first satellite Sputnik-1 in 1957 (it survived for only a few hours in orbit instead of the planned ten days, due to thermal control system issues and the stress of the experience). A Rhesus Monkey was sent in 1949 on another V-2.

Q. Why launchers use several rockets in sequence (multi-stage) instead of just one, or several in parallel?
A. Because sequential burning allows getting rid of the partial containers as they empty, and the mass saving for the on-going stages more than outweighs the initial extra mass of multiple containers (two or three stages are used; beyond that, the complexity overcomes the saving).

Q. Why rockets fly so fast?
A. Because we want high propulsion efficiencies (ratio of useful power to available power), which is very low at low speeds (relative to the exhaust speed). Launchers may appears to climb slowly up after lift-off, but they are accelerating at about 5 \( \text{m/s}^2 \), so that after 5 s they travel at \( v=at=25 \text{ m/s} \) (90 \( \text{km/h} \)).

Q. Why rocket exhaust is so fast?
A. Because we want high thrust with small propellant expense, and \( F \approx m_p v_e \) (in fact, exit speed is usually highly supersonic).

Q. What requires more energy, lifting an object to orbital altitudes (e.g. 300 km) or making it go around?
A. Much more energy to go around (at LEO, \( v \approx 8 \text{ km/s} \), so that kinetic orbital energy is \( E_k/m=\frac{1}{2}v^2=\frac{1}{2} \cdot 8000^2=32 \text{ MJ/kg} \) than to go up (potential orbital energy is about \( E_p/m=g\Delta z=9.8 \cdot 300 \cdot 10^3=3 \text{ MJ/kg} \)).
Q. Why the International Space Station (ISS) is only visible at dawn or at sunset?
A. Because, by daytime, solar radiation dispersed in the atmosphere (sky light) hides the stars and even the Moon; by night, the ISS is so close to Earth that it is under eclipse; hence, we can only see the ISS when there is no daylight but the ISS has not yet enter into eclipse (because we see the solar reflection on it, like for Venus, referred to for that reason by ancient cultures as the Morning Star or Evening Star; Mercury, Mars, and Jupiter can also appear as evening or morning stars, but Venus is by far the brightest).

**ROCKET TECHNOLOGIES**

A rocket engine needs mass to be ejected and an energy source to force the ejection, and with that it yields thrust. The most important variable in rocket technology is the exhaust speed, \( v_e \):

- Mass flow rate \( \dot{m} = \rho v_e A_e \) is proportional to \( v_e \), and this might erroneously point to a low exit speed to minimise propellant expense.

- Thrust \( (F = \dot{m}v_e + A_e (p_e - p_0)) \approx \dot{m}v_e \) is proportional to \( v_e^2 \), and hence the larger the exit speed, the better. The specific impulse is \( I_{sp} \equiv F/(\dot{m}g) \equiv v_e g \), with \( g=9.8 \text{ m/s}^2 \) (see **Propulsion fundamentals**).

- Power consumed \( (\dot{W}_e = \frac{1}{2} \dot{m}v_e^2) \) as a minimum, i.e. neglecting loses) is proportional to \( v_e^3 \), and this poses the limit to achievable exit speed. For instance, if a gas at temperature \( T \) in the rocket chamber expands in the nozzle to vacuum isentropically \( (T \to 0) \), the exit speed is limited to \( v_e = \sqrt{2c_p T} \) (from the energy balance \( c_p \Delta T = \frac{\Delta v^2}{2} = \frac{1}{2}v_e^2 \)), which applies to both a cold-gas rocket (e.g. for nitrogen at 300 K, with \( c_p=1000 \text{ J/(kg·K)} \), it yields \( v_e = \sqrt{2 \cdot 1000 \cdot 300} = 770 \text{ m/s} \), and to hot-gas rockets (e.g. combustion reaction \( \text{H}_2 + \frac{1}{2}\text{O}_2 = \text{H}_2\text{O} \), with adiabatic combustion temperature \( T \approx 3500 \text{ K} \) for the usual fuel-rich mixture, and taking a hot-steam average value of \( c_p \approx 3000 \text{ J/(kg·K)} \), it yields \( v_e = \sqrt{2 \cdot 3000 \cdot 3500} = 4600 \text{ m/s} \)). To go beyond that speed with own-propellant power, nuclear reactions are required.

According to engine size, rocket technologies can be grouped as high power or low power (or thrust, \( F \), since \( \dot{W}_{prop} = Fv_0 \), and it is gravity force which is preponderant in space; by the way, we always use \( g \) for gravity acceleration on the Earth surface, \( g=9.8 \text{ m/s}^2 \):

- High thrust \( (F > m_{craft} g) \), or better high thrust to engine-weight ratio, \( F/W_{eng} = \dot{F}/(m_{eng} g) \), as required for lift-off, orbit injection, landing under vacuum, or rapid manoeuvring. Presently, all high-thrust rockets are of chemical type. They use a lot of fuel (up to 2 000 t) at very high flow rates (may exceed \( \dot{m} = 10 \text{ t/s} \)), releasing huge amounts of energy (exceeding by far those of nuclear power plants). Within the atmosphere (e.g. at lift-off) high-density propellants are preferred (e.g. solid, or kerosene/LOX combination), whereas outside the atmosphere, LH2/LOX is the best mix because it has 30 % more specific impulse. Exhaust speed is limited by the energy balance to \( v_e = \sqrt{2c_p T_{ad}} = \sqrt{2 \cdot 3000 \cdot 3500} = 4600 \text{ m/s} \) for hydrogen combustion with pure oxygen (some 3500 m/s for other propellants), and the limit in thrust \( (F = \dot{m}v_e) \) comes from the maximum burning rate, \( \dot{m} \), which can be made very large by increasing the burning area in solid rockets or by using large **turbopumps** in liquid rockets. Thrust to engine-weight ratio may be in the range \( F/W = 2..3 \) for solid rockets (including propellants), and in the range \( F/W = 10..100 \) for liquid rockets (propellants excluded).
• Low thrust ($F \ll mg$), or better high specific impulse is $I_{sp} = F / (mg)$ (i.e. low thrust specific fuel consumption, TSFC). They use little amounts of fuel (requiring large exhaust speeds, $v_e \gg v_0$, to provide sizeable thrust), and are used either for short-duration thrust, or for small but very long-lasting thrust (e.g. deep-space probes, drag compensation, flight in formation); any kind of rocket (cold-gas, chemical, electromagnetic) can be used for short-pulse thrusters, but, for very-long working periods, only electromagnetic thrusters are suitable, with their minute mass ejection and very-high exhaust speed, which is unlimited (might approach the speed of light), although thrust is limited by available power, $F = m v_e = 2 \eta \dot{W} / v_e$, where the energy efficiency in converting the primary power source to kinetic power ($\dot{W} \equiv \dot{m} v_e^2$), $\eta \equiv \dot{W}_e / \dot{W}$, may be up to $\eta = 0.6$. Presently, maximum power from solar arrays is around $\dot{W} = 10$ kW, so that, even if exit speed is typically 20 000 m/s (instead of the 3000..4000 m/s for chemical ones), it means that available thrust is very low, $F = 2 \eta \dot{W} / v_e = 2 \cdot 0.4 \cdot 10^4 / (2 \cdot 10^4) = 0.4$ N. Thrust to engine-weight ratio are usually $F/W < 0.01$.

There are, however, missions which might be done with both types, like orbit corrections, end-of-life deorbiting, low-Earth-orbit to geostationary-Earth-orbit transfer (LEO-to-GEO), space pushers (to change the orbit of another satellite or debris)... In the case of LEO-to-GEO transfer with chemical rockets, one impulse in low altitude orbit followed by a second impulse in high orbit to make the orbit circular complete the job in a few-hours' time, whereas with electric propulsion the same mission may take weeks with a trajectory that spirals up from the low altitude orbit to the high altitude orbit, consuming only a fraction of the propellant mass used by chemical rockets (which may be some 1000 kg), and this saving is amplified by corresponding launch savings (however, the satellite would receive strong doses of radiation as it spends considerable time crossing Earth's radiation belts; see Space environment and Spacecraft missions).

According to the type of force accelerating the propellant exhaust, rocket technologies can be grouped in:

- Gas-dynamic engines, where propellants (inert or reactant) are pushed backward by pressure forces.
- Electro-dynamic engines, where propellants are pushed backward by electrical or magnetic forces.

Some rocket data (types, size, use...) are presented aside, and compared with other Aerospace engines.

According to the energy type involved in forcing the mass-ejection process, rocket technologies may be grouped in:

- Internal-energy rockets, i.e. where the energy source is stored in the propellants themselves:
  - Mechanical. Direct ejection of a stored fluid by pressure forces (fluid-dynamic acceleration). A cold gas stored at high pressure is sometimes used in small space thrusters, but liquid ejection, or vapour ejection from a heated liquid might be included here.
  - Chemical. Hot gas ejection, generated by chemical reaction, and fluid-dynamic acceleration. Chemical rockets carry one or more substances that produce a hot gas by exothermic chemical reactions (deflagrating at subsonic speed, not detonating; see Combustion kinetics); the gas expands in a nozzle and is ejected at high speed, producing thrust. Most rockets are of chemical type, from small 1 N hydrazine thrusters to the 35 MN of Saturn V. Launchers are all of chemical type because rocket-thrust must be greater than vehicle-weight. There are different types of chemical rockets:
- Solid propellant rockets (developed in the 1100s in China, using black powder).
- Liquid propellants (using monopropellants like $\text{N}_2\text{H}_4$, or bipropellants like kerosene/LOX).
- Hybrid rockets, using propellants in two different states of matter, one solid and the other either liquid or gas (e.g. SpaceShipOne); they are slightly more complex than solid rockets, but safer, controllable, and with almost the same efficiency as hydrocarbon-based liquid rockets.
  - Nuclear. Very hot plasma ejection, generated by nuclear reactions directly into the propellant (e.g. the Fusion Driven Rocket, FDR, being studied by NASA), not requiring conversion to electricity.
- External-energy rockets, i.e. where the energy source is external to the propellants, e.g. is received from the Sun, or from an on-board radioisotope heating units (RHU), which may be used heat the propellants in any of the above rocket types, or be converted to electricity as in photovoltaic panels or in radioisotope thermoelectric generator (RTG), and used to eject the propellants:
  - Electrical. There are two different types of electric rockets:
    - Electrothermal: hot gas generation by electrical energy, and fluid-dynamic acceleration.
    - Electrodynamic: hot or cold plasma generation and acceleration by electrical or electromagnetic forces.
  - Nuclear. Very hot plasma ejection, generated from a stored propellant by nuclear heating (the nuclear material is not ejected). Most nuclear rockets being considered are based on the old NERVA program (Nuclear Engine for Rocket Vehicle Application), a nuclear thermal rocket developed by NASA and terminated in 1972, which used LH2 as propellant, with chamber pressure of 7 MPa, temperature of 2700 K, exit velocity of 8500 m/s and thrust up to 300 kN, with a thrust to weight ratio of 3.4.

Plasma accelerated nuclear rockets might yield $F=10$ MN of thrust with a core reactor of 200 GW, ejecting 250 kg/s of H2 with $I_{sp}=4000$ s ($v_e=40$ km/s). The energy of nuclear reactions should be communicated into the propellant without first being converted to electricity, because it would be almost impossible to release the large amount of low-temperature heat involved in the thermodynamic conversion.

To date, no nuclear rocket has flown or is under construction, but, without nuclear energy, specific energy of propellants is limited to the 13 MJ/kg of LH2/LOX chemical rockets, whereas natural decay of Pu-238 already yields $0.2 \cdot 10^{12}$ J/kg, fission of $^{235}\text{U}$ may yield $80 \cdot 10^{12}$ J/kg, and fusion of deuterium-tritium might reach $360 \cdot 10^{12}$ J/kg. It seems that nuclear rockets will be needed for human flight to Mars, if the trip is limited to few months per journey (otherwise, the radiation dose may be unbearable for the astronauts). NASA plans a first manned trip by 2035, with 180 days going, 400 days there, and 180 days back. With present radiation-shield technology, crewmen will get 1 Sv dose, just above the life-limit for professional workers (the estimated risk of developing a cancer with that dose is around 5%).

All rocket propellants are hazardous materials. Most chemical-rocket chambers are pressure vessels, typically working at 1..25 MPa. Besides, the large temperature jump $\Delta T$ across the wall creates a
compressive stress on the inside and a tensile stress on the outside (within the elastic limit, the stress $\sigma$ is $\sigma=2\alpha E\Delta T/(1-\nu)$, where $\alpha$ is the thermal expansion coefficient, $E$ Young modulus, and $\nu$ Poisson ratio, but the elastic limit is often exceeded in one-shot applications).

In a system with the very large energy density and small mass density like a chemical rocket, coupling between gas dynamics and structural dynamics may yield unwanted vibrations of low or high frequency (when compared with the sound speed divided by rocket length); e.g. the instability called pogo oscillation may develop in liquid-propellant rockets when a surge in engine pressure increases back pressure against the fuel coming into the engine, reducing engine pressure, causing more fuel to come in, and increasing engine pressure again.

It should be noted that, besides chemicals being used for propulsion, pyrotechnics are also used as igniters of main propellants, separation of rocket states, payload fairing, payload components, and several abnormal events (fire suppression, payload rescue, rocket destruction...); a complex launcher may have several hundred energetic devices.

**Cold gas rockets**

Mechanical propulsion by cold gas ejection (from a gas reservoir of N\(_2\), or better He, or from gasification from a liquid reservoir of pressurised butane, ammonia, or hydrazine), is used in attitude control and small orbit-corrections. The storage tank is connected via a control valve, to a plenum chamber, and let to escape through a nozzle. Cold gas rockets are the most reliable, but have very low thrust, and are expensive in mass load. The Manned Manoeuvring Unit (MMU) that astronauts use for EVA in the 1980s, had two aluminium tanks (with kevlar wrappings) containing 6 kg of N\(_2\) each, and 24 many small thrusters.

Maximum exit speed can be estimated by energy conservation from rocket chamber to nozzle exit, $c_p T + v_e^2/2 = \text{constant}$; e.g. for N\(_2\) stored at $T_c=300$ K, $v_e = \sqrt{2c_p T_c} = \sqrt{2 \times 1000 \times 300} = 770$ m/s ($I_{sp}\approx\sqrt{v_e/g}=79$ s). Notice that efficiency ($v_e$ or $I_{sp}$) grows with the square root of chamber temperature $T_c$ and decreases with molar mass, $M$, since $c_p,\text{molar}$ is almost a constant (e.g. $c_p,\text{molar}=5R/2=21$ J/(mol·K) for monoatomic gases, $7R/2=29$ J/(mol·K) for monoatomic gases), and $c_p=c_p,\text{molar}/M$; in this respect, notice that contrary to $c_p,\text{molar}$, there are large differences in $c_p$ (i.e. in mass thermal capacity) among different gases; e.g. at say 300 K, $c_p(\text{H}_2)=14000$ J/(kg·K) and $c_p(\text{Xe})=160$ J/(kg·K). Cold-gas rockets can be enhanced by heating the gas to be expelled, e.g. by electrical energy (as in Electro-thermal rockets), by nuclear energy (difficult to implement, but with potentially high thrust, >1 MN, and very high specific impulse, up to $I_{sp}=1000$ s, $v_e=10000$ m/s, with H\(_2\)), or by an external energy source (e.g. solar heating), but the problem of high heat-transfer-rates to the gas has not yet been solved. Chemical rockets too, may take advantage of this additional energy source, but in practice it is only used on medium-temperature chemical rockets like those using hydrazine monopropellant.

**Solid propellant rockets**

Solid propellant rockets (usually shorted to 'solid rockets') are simple and reliable systems that can provide great thrust, but cannot be stopped and re-started, neither controlled in intensity (only in direction with a gimbaled nozzle), and are less efficient than liquid-propellant rockets: solid propellants have heating values
10..20 % of liquid fuel values, and the exhaust carries a lot of solid particles, a visible plume that may be
dark or white (although there are some smokeless powders). Solid propellants are used in boosters, insertion
and apogee engines, descent stage retrorockets, aircraft ejection seats, and pyrotechnics. Ignition can be
started in several ways (see igniters); large solid rockets typically use a two-stage ignition system: first, a
small electrically-triggered pyrotechnic device (initiator) starts the main igniter, and then this main igniter
(booster), which is really a small solid rocket that burns for a few tenths of a second, generates a hot-gas
spear that ignites the whole surface of main propellant.

The processes the solid-propellant follows on burning are: powder heating by conduction through the hot
burning gas, powder decomposition (pyrolysis) near the surface, vaporization (sometimes preceded by
melting, but there should be no charring), thorough mixing of different gases from the different propellant
components, and gas-phase reaction (the flame is very close to the surface, <1 mm). The combustion
chamber in solid rockets is not cooled because the propellant isolates the wall except at the nozzle, where
one relies on ablative cooling (near the nozzle-throat, reinforced materials are used to avoid too much
ablation).

Solid-rocket performances basically depend on the type of propellant, and on grain geometry (how the
propellant is laid out). Most solid rockets work within the atmosphere, below 50 km altitude, so that the
minimum environmental pressure is >100 Pa ($p_{ISA,50km}=80$ Pa) and their nozzles do not have to be design
for vacuum conditions. Solid propellants may be:

- Monopropellants. A single compound, usually in granular form. The most used nowadays is
  ammonium perchlorate (AP), followed by ammonium nitrate (AN), which is less expensive and less
  contaminant, but less energetic.

- Composite propellants. Heterogeneous mixture of fuel and oxidiser powders glued within a binder
  (e.g. the original black powder, and the modern ammonium perchlorate composite propellant,
  APCP).

- Double-base propellant. Homogeneous grain, usually nitrocellulose dissolved in nitroglycerine and
  solidified. Very dangerous.

**Ammonium perchlorate (AP)**

Ammonium perchlorate ($\text{NH}_4\text{ClO}_4$ or AP) is the most used solid propellant, either alone (as
monopropellant), or as oxidiser in composite propellants (it has 60 % mass content in oxygen). AP is
produced by reaction between ammonia and perchloric acid.

Physical properties of AP: $M=0.1175\ \text{kg/mol}$, $\rho=1950\ \text{kg/m}^3$, $c_p=1460\ \text{J/(kg·K)}$, $k=0.42\ \text{W/(m·K)}$, soluble
in methanol, solubility in water 0.12 kg/L at 0 ºC, 0.21 kg/L at 20 ºC, 0.57 kg/L at 100 ºC, with the normal
form of a white crystalline salt powder (typical particle size is 20..200 µm; it is classified as explosive if
particle size <15 µm). It decomposes before melting.

Thermochemical properties of AP: standard enthalpy of formation $h_f=-295\ \text{kJ/mol}$, standard Gibbs-
function of formation $g_f=-89\ \text{kJ/mol}$, standard absolute entropy $s=184\ \text{J/(mol·K)}$, thermal decomposition
starts at $T_{\text{decomp}}=243\ ^\circ\text{C}$ with an endothermic solid-phase transition (from orthorhombic to cubic); at about
330 °C a slow exothermic decomposition starts, generating toxic fumes, \( \text{NH}_4\text{ClO}_4(s) = \text{NH}_3(g) + \text{HClO}_4(g) + 98 \text{ kJ/mol} \), but with little advance (except if large bulk amounts make the evolution adiabatic and cause a runaway, i.e. autoignition). This chemical reaction in solid phase is not common in other propellants, which just melt or vaporise before reaction. If temperature increases, at about 460 °C a quick deflagration takes place, and the temperature jumps to about 2400 K; the chemical kinetics depends a lot on AP purity (minute impurities act as catalysers), on environmental conditions (the presence of air, its pressure, light, vibrations...), and on geometrical details (particle size, sample size, test-cell size...), and a much lower autoignition temperature is often quoted (\( T_{\text{autoign}} = 240 \text{ °C} \)); the flash temperature of AP in air seems to be of that order too (no data found). The following stoichiometry may apply to AP deflagration:

- **Complete redox reaction:** \( \text{NH}_4\text{ClO}_4 = 2\text{H}_2\text{O} + 0.5\text{Cl}_2 + 0.5\text{N}_2 + 189 \text{ kJ/mol} \) (\( h_{\text{LHV}} = 189/0.1175 = 1.61 \text{ MJ/kg} \)), a limit in practice.

- **Typical reaction at operating pressure in rocket chambers** (\( \approx 10 \text{ MPa} \); hydrogen chloride appears): \( \text{NH}_4\text{ClO}_4 = 1.5\text{H}_2\text{O} + 1.25\text{O}_2 + \text{HCl} + 0.5\text{N}_2 + 163 \text{ kJ/mol} \), corresponding to a \( h_{\text{LHV}} = 163/0.1175 = 1.39 \text{ MJ/kg} \). Notice that this heating value increases a lot when additional fuels take advantage of the excess oxygen (e.g. APCP may have \( h_{\text{LHV}} > 4 \text{ MJ/kg} \) instead of the 1.4 MJ/kg of pure AP). To get rid of the environmental problem created by chlorine and hydrogen chloride, ammonium nitrate (AN, \( \text{NH}_4\text{NO}_3 \)), and ammonium dinitramide (ADN, \( \text{NH}_4\text{N(NO}_2)_2 \)) are candidates to substitute ammonium perchlorate.

- **Typical at laboratory conditions** (100 kPa, low heating rate; nitrous oxide appears): \( \text{NH}_4\text{ClO}_4 = 2\text{H}_2\text{O} + 0.5\text{Cl}_2 + 0.875\text{O}_2 + 0.25\text{N}_2 + 0.25\text{N}_2\text{O} + 167.4 \text{ kJ/mol} \) (i.e. the lower heating value is \( h_{\text{LHV}} = 167.4/0.1175 = 1.42 \text{ MJ/kg} \)), with an adiabatic temperature of 2400 K.

Several major accidents have occurred at AP-manufacturing premises and during AP-transportation, as the **Pepcom disaster** of 1988-05-04, where about 1500 t of AP exploded (there were 4000 t AP in total at the plant) causing 2 deaths, 372 people injured, and an estimated US$100 million of damage; and it was related to space propulsion because that AP was to be used in the Shuttle's Solid Rocket Boosters (SRB), and the accumulation was due to NASA halting Shuttle flights after Challenger's accident of 1986-01-28. What happened was that Pepcom employees were repairing a steel framed structure with a welding torch, and some sparks started a small fire on adjacent fiberglass material (there was some residue of AP nearby, but it was known that sparks could not ignite AP powder at room conditions; after almost half an hour trying to suffocate the fiberglass fire (the AP residue had joined in burning), the flames reached a barrel of AP (drum-shape containers of 200 L) that exploded, spreading the fire; several minutes later, the fire reached one of the main storage area where aluminium containers with 2.5 t of AP were stoked ready for shipping to **Morton Thiokol**, the SBR manufacturer (each SRB is loaded with 550 t of APCP, i.e. 385 t of AP); the two major blasts measured 3.0 and 3.5 in the Richter scale.

**Ammonium perchlorate composite propellant (APCP)**

AP is not used alone (it is a loose crystalline powder) but as a consistent solid by using an elastomer binder, and often with some additional fuel as in APCP. The binder may be hydroxyl-terminated polybutadiene (HTPB, a polymer of relative composition \( \text{CH}_{1.54}\text{O}_{0.007} \) with a molar mass around \( M = 2.5 \text{ kg/mol} \), with \( \rho = 950 \text{ kg/m}^3 \), \( c_p = 2800 \text{ J/(kg·K)} \), \( T_m = 240 \text{ °C} \)), poly-butadiene acrylonitrile (PBAAN), carboxy-terminated poly-butadiene (CTPB)..., and the additional fuel is usually a metal powder (Al, Fe, Mg...), e.g.:
• In Ariane V solid booster, APCP composition (by mass) is 68 % AP, 18 % Al powder, and 14 % HTPB binder.
• In Shuttle solid booster, APCP composition (by mass) is 70 % AP, 16 % Al powder, and 14 % HTPB binder.

APCP is **cast into shape** by curing HTPB with isocyanates or epoxies to form cross-linked networks, as opposed to powder pressing in former propellants. APCP cost is over 50 €/kg (much higher than bulk cryogenic propellants; e.g. LH2/LOX may cost 5 €/kg). When burning, gas temperature inside the chamber may go over 3000 K (up to 3300 K), with pressure up to 10 MPa, but temperature at the burning solid surface hardly reaches 1000 K. The burn rate is heavily dependent on mean AP particle size as the AP absorbs heat to decompose into a gas before it can oxidize the fuel components. Common APCP formulations call for 30..400 µm AP particles and 2..50 µm Al particles, which sit at interstitial positions of the former. The flame temperature around Al particles (with oxygen released by AP) is up to 4100 K, and the large amount of energy released quickly melts the raw solid composite, increasing chamber pressure and exit speed. Al particles first melt into blobs that break down to \(d<500\) µm droplets being carried on with the gas flow while burning, suppressing acoustic instabilities; besides, a smoke of Al\(_2\)O\(_3\) particles (some \(d<5\) µm in size) forms near the burning surface and gets denser downstream.

The exhaust from APCP solid rocket motors contains gases (mostly water vapour, carbon dioxide, hydrogen chloride), and solid particles of metal oxide (Al\(_2\)O\(_3\), melting temperature \(T_m=2300\) K). Notice that, for an APCP with 20 % Al powder, the mass fraction of Al\(_2\)O\(_3\) particles in the exhaust may reach 40 %, which being solid (\(T_m=2300\) K is well above exhaust temperature), does not expand, lowering the specific impulse. The hydrogen chloride can easily combine with water vapour to yield highly-corrosive hydrochloric acid, damaging launch equipment and biasing the pH of local waters and rainfall.

APCP typical properties are: \(\rho=1800\) kg/m\(^3\) \((\alpha=110\cdot10^{-6}\) 1/K\), \(c_p=1460\) J/(kg·K), \(k=0.5\) W/(m·K). The adiabatic temperature is \(T_{ad}=3000\) K (may reach \(3300\) K). The specific impulse of APCP is around \(I_p=250\) s \((v_e=2450\) m/s\) at sea-level, and up to \(I_p=300\) s \((v_e=2940\) m/s\) under vacuum. Thrust may range from \(F=50\) N in small rockets, to \(F=12.5\) MN in each of the two solid booster rockets, SBR, used in the Space Shuttle.

If a ponderal formula \(\text{C}_2\text{H}_4\) is used to approximate the cured HTPB, a suitable stoichiometry may be \(6\cdot\text{NH}_4\text{ClO}_4(s)+4\cdot\text{Al}(s)+2\text{C}_2\text{H}_4=2\cdot\text{Al}_2\text{O}_3(s)+6\cdot\text{HCl}(g)+12\cdot\text{H}_2\text{O}(g)+2\cdot\text{CO}_2(g)+2\cdot\text{CO}(g)+\text{H}_2(g)+3\cdot\text{N}_2(g)\), although the mass percentage (of AP/Al/HTPB) here is 81/12.5/6.5 instead of the more common 68/18/14. The heating value of typical APCP formulations is around 4 MJ/kg, which may be compared with the almost 10 MJ/kg for RP1/LOX propellant. Though increasing the ratio of Al-powder to AP up to the stoichiometric point increases the combustion temperature (the stoichiometry \(6\cdot\text{NH}_4\text{ClO}_4+8\cdot\text{Al}=4\cdot\text{Al}_2\text{O}_3+3\cdot\text{Cl}_2+12\cdot\text{H}_2\text{O}+3\cdot\text{N}_2\), with 23 % of Al by mass, would yield \(h_{LHV}=8.5\) MJ/kg and \(T_{ad}<4000\) K), the larger fraction of Al\(_2\)O\(_3\)(s) precipitating from the gaseous solution creates globules of solids or liquids that slow down the flow velocity as the mean molecular mass of the flow increases. In addition, the chemical composition of the gases change, varying its heat capacity; usual values at the exhaust are \(c_p=1900\) J/(kg·K), \(\gamma=1.2\), \(M=0.026\) kg/mol, \(k=0.2\) W/(m·K). Because of these phenomena, there

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exists an optimal non-stoichiometric composition for maximizing $I_{sp}$ of roughly 16% by mass, assuming the combustion reaction goes to completion inside the combustion chamber.

**Liquid propellant rockets**

Liquid rockets are used in boosters, insertion, apogee, orbit control, and attitude control. They can provide large thrust, and can be easily controlled (can be restarted). Liquid rockets are typically more powerful and efficient than solid rockets, but are also more complex and difficult to store. The first flight of a liquid-propellant rocket was in 1926, by Goddard, using liquid oxygen and gasoline. Pumping the propellants from their tanks to the combustion chamber is performed by turbopumps driven by the exhaust gases from an auxiliary burner, although electric pumps battery-powered can be used in small rockets.

Chamber temperature are very high $T_c=2800..3800$ K, implying gas dissociation, and requiring active cooling, usually regenerative cooling (i.e. with heat recovery); the fuel is preferred as coolant because of the higher thermal capacity (a coiled tubing surrounds the combustion chamber and nozzle). Maximum wall temperature is 1000..1200 K, with some 200 K drop across the wall. The specific impulse is $I_{sp}=310..450$ s, with exit speed $v_e=3000..4400$ m/s. Thrust may range from $F=0.5$ N in small hydrazine thrusters to $F=1.4\cdot10^6$ N in Shuttle Main Engines (SME). Liquid propellants may be a single substance that decomposes, or two substances (a fuel and an oxidiser) that mix and react in the combustion chamber.

A problem of spacecraft propulsion with liquids (and with any other liquid-reservoir in space) arises on partially-filled tanks under microgravity: how to force liquid (and only liquid), to go out when needed. There are many ways to solve the problem (other than use flexible containers without ullage, like in a bladder): one way is to create some directional acceleration (by rotation or a small thrust) to position the liquid at the opening (but this introduces perturbations on the rest of the spacecraft); a better way is to rely on surface-tension forces to retain the liquid and displace the vapours and gases, feeding the output line passively by capillarity without any external involvement. There are several types of capillary devices that may be used to 'fix the liquid', from simple wettable vanes to wicks (like in heat pipes).

For a given bipropellant pair, molar mass, thermal capacity, combustion temperature (and hence specific impulse) depend on the ratio of oxidizer mass to fuel mass (known as mixing ratio, $r_m = \dot{m}_o / \dot{m}_f$). For a given chamber-pressure, there is a mixing ratio that makes specific impulse a maximum (Fig. 1).
Fig. 1. Specific impulse $F_{sp} = F / \dot{m}_p = F / (\dot{m}_O + \dot{m}_F)$ (i.e. thrust divided by mass-flow-rate of propellants), in N/(kg/s)=m/s), as a function of oxidiser/fuel mixing ratio (O/F), $r_{OF} = \dot{m}_O / \dot{m}_F$, for several liquid bipropellants (Haidn-2008, DLR).

Common bipropellant combinations are:

- **LH2 and LOX**, named LH2/LOX or LH2/LOX (i.e. liquid hydrogen with liquid oxygen), as used in Saturn V (2nd and 3rd stages), Space Shuttle Main Engines, Ariane 5 Vulcain... This combination is the most efficient for chemical rockets (Fig. 1): it has $F / \dot{m}_p = v_e=4500$ m/s ($I_{sp}=460$ s), but has a handicap on the very low density of hydrogen (71 kg/m$^3$), and its very low temperature (20 K). The oxidiser/fuel mass ratio used is around $r_{OF}=6$ to maximise specific impulse (Fig. 1), instead of stoichiometric ($r_{OF}=8$; $H_2+\frac{1}{2}O_2=H_2O$) to lower the molar mass of the exhaust from $M_{H2O}=0.018$ kg/mol to $M=0.010$ kg/mol (due to the 25 % excess H$_2$, and up to 3% of OH and up to 3% of H formed; O$_2$ and O are below 0.1%); as a consequence, chamber temperature decreases from about 3750 K to about 3500 K. Chamber pressure is around 10 MPa. The ratio chamber volume over throat area is in the range $V/A=0.7..1.5$ m. Cryogenic liquids are fed to the main combustion chamber through several-hundred injectors, by means of separate turbopumps driven by hot fuel-rich gases from a preburner; oxygen enters as a cryogenic liquid at around 10 m/s, but hydrogen enters as a hot gas (with some water-vapour from the preburner, at some 100..300 m/s. USA first used LH2/LOX engines in 1963, followed by Japan in 1977, France in 1979, China in 1984, Russia in 1987 and India in 2014. The largest LH2/LOX rockets are the RS-68 of more than 3 MN each, used in Delta IV Heavy (following the RS-25 of almost 2 MN each, used on the Shuttle; see below).

- **Kerosene and LOX** (also named RP1/LOX, where RP stands for rocket propellant), as used in 1st stage of Soyuz, Saturn V, Energia, Delta, Atlas, Falcon 9 (the first privately-owned rocket, which carried SpaceX-Dragon cargo to the ISS in 2010), Electron... Specific impulse is lower than for LH2/LOX, but kerosene is much cheaper and denser ($\rho_{RP1}=820$ kg/m$^3$ against $\rho_{LH2}=71$ kg/m$^3$), and more handy (not cryogenic). Typical oxidiser/fuel mass-ratio used is in the range $r_{OF}=2.3..2.7$ to maximise specific impulse (Fig. 1), in spite of stoichiometry being at $r_{OF}=3.4$.
(C_{12}H_{24}+18O_2=12CO_2+12H_2O). Adiabatic combustion temperature may reach 3600 K, with a mean thermal capacity ratio of expanding gases of $\gamma=1.24$, and exit velocity at sea level $v_e=3500$ m/s ($I_{sp}=360$ s under vacuum, $I_{sp}=320$ s at sea level). The ratio of chamber volume to throat area is in the range $V/A=1..2.5$ m. The liquids are fed by means of turbopumps driven by hot oxidiser-rich gases from a preburner (one turbine may drive the two propellant's compressors because their densities are similar). The largest RP1/LOX rockets are the Russian RD-171 of 17.5 MN each; those, and the RD-180 (used in Atlas launchers), work with a high pressure (25 MPa) in the combustion chamber.

- Other alkane fuels and LOX (e.g. LNG/LOX, using liquefied natural gas or methane with $\rho=430..660$ kg/m$^3$; LPG/LOX, using liquefied petroleum gas of $\rho=570..600$ kg/m$^3$; C$_3$H$_6$/LOX, using propylene of $\rho=614$ kg/m$^3$). Japan has developed a 100 kN LNG/LOX rocket (LE-8), thought to work with an oxidiser-to-fuel mass ratio of $r_{OF}=3.5$ (see Fig. 1) and exit speed of $v_e=3600$ m/s ($I_{sp}=370$ s in vacuum). SpaceX Raptor uses subcooled liquid methane and LOX.

- Maximum exhaust speed is about $v_e=3370$ m/s ($I_{sp}=344$ s in vacuum), with a range of $F=10$ N to $F=500$ kN (e.g. $F=130$ kN in Shuttle RCS).

- N$_2$H$_4$ (or derivatives: MMH or UDMH) and N$_2$O$_4$, i.e. hydrazine, or hydrazine compounds, with liquefied dinitrogen-tetroxide. These bipropellants are easily stored at room conditions in liquid state (at 25 °C, N$_2$O$_4$ needs $p>120$ kPa, its normal boiling point being $T_b=21$ °C), they are hypergolic (i.e. they ignite spontaneously on contact with each other, requiring no ignition system), and the rocket can easily be stopped and re-started (but are not good for short pulsing), with typical thrust range from 10 N to 500 kN. Hydrazine, monomethyl-hydrazine (MMH), and unsymmetrical-dimethyl-hydrazine (UDMH), have similar propulsive properties, with UDMH a bit lower than MMH and this a bit lower than N$_2$H$_4$ (e.g. specific impulse under vacuum for UDMH, MMH, and N$_2$H$_4$, are 325 s, 330 s, and 340 s, respectively), though UDMH is not suitable to nozzle cooling because of its lower thermal stability. The oxidiser/fuel mass ratio for UDMH, MMH, and N$_2$H$_4$, are 2.0, 2.3, and 2.4, respectively, with adiabatic temperatures of 3200 K, 3250 K, and 3300 K, respectively. Exhaust gases have a mean molar mass of about $M=0.020$ kg/mol, and $\gamma=1.30$. Mixture of UDMH/N$_2$H$_4$ have been used with special names; e.g. 50/50 was Aerozine-50 (used in Apollo LM), 75/25 was UH-25. The basic advantage of MMH and UDMH over N$_2$H$_4$ (e.g. specific impulse under vacuum for N$_2$H$_4$, MMH, and UDMH, are 325 s, 330 s, and 340 s, respectively) though UDMH is not suitable to nozzle cooling because of its lower thermal stability. The oxidiser/fuel mass ratio for UDMH, MMH, and N$_2$H$_4$, are 2.0, 2.3, and 2.4, respectively, with adiabatic temperatures of 3200 K, 3250 K, and 3300 K, respectively. Exhaust gases have a mean molar mass of about $M=0.020$ kg/mol, and $\gamma=1.30$. Mixture of UDMH/N$_2$H$_4$ have been used with special names; e.g. 50/50 was Aerozine-50 (used in Apollo LM), 75/25 was UH-25. The basic advantage of MMH and UDMH over N$_2$H$_4$ is the wider liquid range (N$_2$H$_4$ solidifies at $-50$ °C). Dinitrogen tetroxide, N$_2$O$_4$, is often short-named as 'nitrogen tetroxide' or NTO in rocket argot. Modern tendency is to add to NTO a small percentage of nitric oxide to inhibit stress-corrosion cracking of titanium alloys, and in this form, propellant-grade NTO is referred to as "Mixed oxides of nitrogen" or MON; e.g. the Space Shuttle reaction control system (RCS) uses two 130 kN rockets of MMH/MON3 (MON3 is NTO containing 3% of NO by mass). Ariane 5 upper stage Aestus is a 30 kN MMH/NTO rocket. Small MMH/NTO rockets of 10 N are used in orbit and attitude control systems on many spacecraft (e.g. Rosetta has 24 such thrusters). China's manned spacecraft Shenzhou also makes use of MMH/NTO rockets.

- Kerosene and H$_2$O$_2$ (sometimes stated as RP1/HTP, the short-name of high-test peroxide, strong aqueous solutions of hydrogen peroxide with mass concentration over 80 %). HTP is used as propellant in rockets and torpedoes, either as a strong oxidant in bipropellants (RP1/HTP has...
v_e=3300 m/s, I_sp=340 s, a little smaller than RP1/LOX, but denser and not cryogenic), or as a monopropellant (see below).

- Others:
  - Ethanol and LOX were the propellants used in the first space flight (that of V-2 in 1944).
  - NH₃ and LOX were used on XLR-99 rocket engine, propelling the X-15 research craft.

Liquid monopropellants are used in small and medium-size rockets for orbit control, attitude control, using a catalyst in the reaction chamber; most used monopropellants are:

- N₂H₄. **Hydrazine** is the most used (data aside), in spite of being highly toxic and flammable. It has v_e=2100 m/s (I_sp=220 s under vacuum), and typical engine thrust range from F=0.5 N to F=50 kN (Viking). Its decomposition reaction can be set in general as 3N₂H₄=4(1−x)NH₃+(2x+1)N₂+6xH₂, with 0<x<1 being the fraction of ammonia dissociated, which depends on catalyst and resident time. The energy released in decomposition is about six times lower than when oxidising with N₂O₄ in bipropellants, reaching temperatures in the range 800..1500 K (the more NH₃ the hotter). The fuel needs thermal control to avoid freezing (T_f=−2 °C), but it is a simple and reliable system capable of pulsing (∆t≥10 ms, up to 10⁶ pulses); UDMH and MMH are not used as monopropellants because of their greater chemical stability. It is not sensitive to friction or impact. Other monopropellants like hydrogen peroxide and nitrous oxide, have smaller specific impulse: about 1600 m/s with H₂O₂(liq,80 %) and about 1500 m/s with N₂O(liq).

- H₂O₂. High-test **hydrogen peroxide** (HTP) is used on some small attitude-control rockets; e.g. a 90 % weight solution at 20 °C has ρ=1400 kg/m³, c_p=2800 J/(kg·K), T_f=−10 °C, T_b=140 °C, p_v=300 Pa (total), k=0.60 W/(m·K). In contact with a catalyst (permananate, silver...) it decomposes into a mixture of steam and oxygen at almost 900 K (i.e. with no remaining liquid water): H₂O₂=H₂O+½O₂+98 kJ/mol. The specific impulse is I_sp=160 s (v_e=1600 m/s). H₂O₂ was the first liquid monopropellant used: first to drive the steam catapults for V-1 flying bombs, and later to drive the turbopumps that fed fuel (ethanol) and oxidiser (LOX) to the V-2 rockets.

- CH₃NO₂. **Nitromethane** decomposes as CH₃NO₂=CO+H₂O+½H₂+½N₂; it is a popular additive to gasoline engines, particularly in model engines, because it provides additional oxygen (it only requires 1 kg of air per 1 kg of fuel).

- N₂O. **Nitrous oxide**, the “laughing gas” used in Medicine for its anaesthetic and analgesic effects, is a non-flammable, mildly toxic colourless gas, which can be easily stored as a liquid at p=5.2 Pa at 20 °C. With suitable catalysts, it decomposes to form breathing air, releasing a lot of energy: N₂O=N₂+½O₂+82 kJ/mol, which heats the gases to >1500 K (it can yield an I_sp=180 s under vacuum, with exit speed of v_e=1800 m/s). Its global warming potential is 300 times larger than that of CO₂ (GWP=300). The combination of nitrous oxide (N₂O) with hydroxyl-terminated polybutadiene fuel (HTPB) has been used by SpaceShipOne, SpaceShipTwo, and others, with I_sp=250 s (theoretically, I_sp=320 s); two-phase flow in the feed system makes injector design complicated.

A white mist can often be seen near launch vehicles; this is due to atmospheric water-vapour condensation around the cryogenic oxygen gas being vented from LOX tanks (LOX is used in most rocket launches, e.g.
using RP1/LOX or LH2/LOX, and all cryogenic tanks must vent the boil-off gas to avoid pressure build-up. This continuous venting of cryogens makes them unsuitable for in-space propulsion (they are only used in launches from the Earth); propulsion in space requires storable propellants like hydrazine and its derivatives, and the simpler to operate the better (i.e. no ignition devices as in hypergolic mixtures, or catalytic reaction).

In any case, chemical rockets are limited in performance because they only rely on energy stored in the propellants (chemical energy). Higher rocket efficiency (specific impulse) can be obtained by adding external energy to the propellant, e.g. a solar-powered heater in a cold-gas thruster, or a nuclear-powered plasma accelerator.

Exercise 1. Find the heat release in the stoichiometric chemical reactions of:

a) LH2 with LOX.

b) N2H4 with N2O4.

Sol. It depends on the products of the reaction. Assuming complete oxidation of hydrogen (to yield H2O), and no formation of nitrogen oxides (in the second case), the stoichiometry is H2+½O2=H2O, and N2H4+½N2O4=¾N2+2H2O, respectively.

For the given stoichiometry, the heat release depends on the thermodynamic state of the components before and after the reaction. The standard higher heating value assumes an isobaric process with each substance in its pure state at 25 °C and 100 kPa, which in the first case is H2(g)+½O2(g)=H2O(l) and in the second N2H4(l)+½N2O4(g)=¾N2(g)+2H2O(l), because, at 25 °C and 100 kPa, pure water is a liquid (Tm=0 °C, Tb=100 °C), hydrazine is a liquid (Tm=2 °C, Tb=114 °C), and N2O4 is a gas (Tm=−11 °C, Tb=21 °C). Hence, with the standard enthalpy of formation from Thermochemical data, we get in the first case the energy balance 0+½0=−(286)+hHHV→hHHV=286 kJ/mol of hydrogen burned (0.286/0.002=142 MJ/kg of H2), and in the second case 50.6+½9.2=¾0+2·(−286)+hHHV→hHHV=627 kJ/mol of hydrazine. But when both the fuel and the oxidiser are carried aboard, it is more convenient to give heating values by unit of total mass consumed, i.e. in the case of H2+½O2=H2O, we get 286 kJ by the combination of 0.002 kg of H2 with ½0.032=0.016 kg of O2 (0.018 kg in total), hence hHHV=286/0.018=15.9 MJ/kg of propellants; and similarly in the second case: we get 627 kJ by the combination of 0.032 kg of N2H4 and ½0.092 kg of N2O4 (0.078 kg in total), hence hHHV=627/0.078=8.04 MJ/kg of propellants.

But in propulsion we are more interested on the lower heating value, hLHV, i.e. assuming that we are not taking advantage of the condensation of the water-vapour produced at high temperature, so that we use the enthalpy of formation of the ideal vapour state for H2O, −242 kJ/mol instead of −286 kJ/mol, and the above values become: for H2+½O2=H2O, 0+½0=−(242)+hLHV→hLHV=242 kJ per mol of H2 → hLHV=242/0.018=13.4 MJ/kg of propellants; for N2H4+½N2O4=¾N2+2H2O, 50.6+½9.2=¾0+2·(−242)+hLHV→hLHV=539 kJ per mol of N2H4 → hLHV=539/0.078=6.9 MJ/kg of propellants.

Finally, we should take into account that the input reactives are not at the standard state of 25 °C and 100 kPa, where hydrogen, oxygen, and dinitrogen-tetroxide would be gases; they enter as liquids (because they are stored in liquid state for compactness) and thus with lower enthalpy than assumed; for LH2 to change from 20 K (liquid) to 298 K (gas) one has to add, with the perfect-
substance model and data, \( h_{LV} + c_p \Delta T = 448 + 14.2 \cdot (298 - 20) = 4.4 \text{ MJ/kg of H}_2 \) (3.9 MJ/kg with NIST data); for LOX to change from 90 K (liquid) to 298 K (gas) one has to add \( h_{LV} + c_p \Delta T = 213 + 0.913 \cdot (298 - 90) = 0.40 \text{ MJ/kg of O}_2 \) (0.41 MJ/kg with NIST data); for N$_2$O$_4$ to change from 298 K (liquid) to 298 K (gas) one has to add \( h_{LV} = 330 = 0.33 \text{ MJ/kg of N}_2\text{O}_4 \). Hence, per kilogramme of stoichiometric propellant, LH$_2$ with LOX yield 13.4 − (1/9)·3.9 − (8/9)·0.41 = 12.6 MJ/kg, whereas N$_2$H$_4$ with N$_2$O$_4$ yield 6.9·0.59·0.33 = 6.7 MJ/kg (0.59 being the stoichiometric mass fraction of N$_2$O$_4$).

**Electrothermal rockets**

They are electrically-heated gas rockets, i.e. electric heating of a gas and mechanical acceleration from a high chamber-pressure through a gas-dynamic nozzle; thrust is by gas pressure-forces on walls. Internal or external power sources (solar power, batteries, nuclear...) may be used. Notice that if only a fraction of the flowing gas is heated, mixing with the rest of the gas should be accounted for. According to the heating method we may have:

- A 'resistojet', based on a cold-gas rocket, with an electrical heater (fed from the electrical power system) in the chamber preceding the nozzle, what increases the temperature of the gas supplied from storage tank and increases exit speed \( (v_e) \) and specific impulse \( (I_{sp}) \) almost proportionally to the square root of the chamber temperature achieved (see Rocket exit speed, below). Notice that the gas cannot get hotter than the resistor, what limits the maximum temperature to some 2500 K. Resistojets are not only being used in cold-gas rockets but on hydrazine rockets too (they are currently used in the ISS to adjust mission orbit and attitude, and on many GEO satellites). Water might be used for space propulsion if steam is generated in a resistojet, but a lot of specific energy would be required (it may take 2.5 MJ/kg to just get steam at <400 K (obtaining some \( v_e = \sqrt{2c_pT_0} = \sqrt{2 \cdot 2000 \cdot 400} = 1250 \text{ m/s (} I_{sp} = 125 \text{ s}) \), less than half of what would be obtained using that heating on He or H$_2$); besides, there is the potential problem of water freezing.

- An 'arcjet', based on a cold-gas rocket, but instead of adding electrical resistances, the gas is heated in a voltaic-arc between a central electrode and an annular one. In this case, walls remain colder than the arc-heated gas. To avoid electrode erosion by high electric currents, a solenoidal magnetic field is added, creating azimuthal motion. The ARGOS satellite, launched in 1999, tested a 26 kW ammonia arcjet producing a thrust of 2 N with a specific impulse over \( I_{sp} = 800 \text{ s (} v_e = 8000 \text{ m/s)\).}

**Electrodynamic rockets**

They are electrically accelerated plasma rockets, i.e. where plasma generation and acceleration is not achieved by pressure in nozzle-flow, but by electrostatic (Coulomb) or electromagnetic (Lorentz) forces on the electrically-charged gas, with electrodes or with coils. For a particle with mass \( m \) and electric charge \( q \), exposed to an electric field \( \vec{E} \) and a magnetic field \( \vec{B} \), the equation of motion is:

\[
 m \frac{d\vec{v}}{dt} = q\vec{E} + q(\vec{v} \times \vec{B}) + \vec{F}_p \tag{1}
\]

where \( \vec{F}_p \) is a pressure or collisional force. Internal or external power sources (solar power, batteries, nuclear...) may be used. Power is used to ionize the stored propellant (i.e. to create the plasma), and to
create a voltage gradient to accelerate the stream of ions to very high exhaust velocities. Small electric rockets can be powered by solar arrays, but more powerful rockets (or for operation far from the Sun), require nuclear energy at least to create the plasma.

Although thermal rockets (with electric heating or with chemical reactions) are powerful, with a thrust range from mN to MN, their efficiency has an upper limit corresponding to a maximum specific impulse of $I_{sp}=1000$ s ($v_e=10,000$ m/s) or so. It is to overcome this limit that electrodynamic rockets are being designed (although their present thrust range is very much limited because of available power). The two main types of electrodynamic rockets are ion thrusters and Hall thrusters, but many other types are being developed (e.g. VASIMR uses radiofrequency heating and magnetic acceleration). Both for ion thrusters and Hall thrusters a high atomic mass and low ionization potential gas is used, e.g. xenon (Xe, $M=0.131$ kg/mol, first ionization energy 12.1 eV, i.e. 1170 kJ/mol; stored at 15 °C and 6 MPa, has $\rho=1615$ kg/m$^3$).

**Fig. 2. a) Ion thruster (Wiki). b) Hall thruster (Wiki).**

Electric thrusters are better characterised by their energy efficiency, $\eta$ (kinetic energy of exhaust divided by applied electrical power, i.e. current times voltage difference), instead of by specific impulse, $I_{sp}$:

$$
\eta \equiv \frac{\frac{1}{2}mv^2}{IV} = \frac{1}{2} \frac{Fv_e}{IV}
$$

(2)

Russian satellites have used electric propulsion since 1970s, whereas Western electric rockets started in 1990s; most current GEO-satellites incorporate electric thrusters for north-south station-keeping, orbit rising, and end-of-life de-orbiting. The first use of Hall thrusters on lunar orbit was ESA’s lunar mission SMART-1 in 2003. EU manufacturers of electrodynamical thrusters are SNECMA, Astrium, Qinetiq...

**Ion thrusters**

A neutral gas (e.g. Xe) is injected into the chamber and ionised (e.g. by bombarding it with energetic electrons, or by radiofrequency or microwaves ionization); ions escape through a high-voltage positive screen grid (>1000 V) and are accelerated through a negative grid at the exhaust (e.g. at −200 V), whereas electrons trapped in the chamber are conducted through the wall to be injected in the exhaust stream to get
a neutral plasma (neutral on the average, but ions do not recombine to neutral molecules because of the highly different speeds), and thus preventing a very rapid negative charging of the spacecraft.

About 10 % of the power is used to create the plasma, and 90 % to accelerate it; a 1 keV acceleration of a Xe-ion corresponds to 38 km/s exit speed. Notice however that, when properly operating, the accelerator grid should collect no ions or electrons, and hence its power supply should consume no power, only apply a static voltage. On the other hand, the power supply connected to the neutralizer must pass an electron current equal in magnitude to the ion beam current, and must also have the full accelerating voltage across its terminals; it is therefore this power supply that consumes (ideally) all of the electrical power in the device. The energy-conversion efficiency is around 90 %.

The electrostatic field stores energy by unit volume in the amount \( u = \frac{1}{2} \varepsilon |E|^2 \), which is equivalent to an electrostatic pressure, \( p = u \), and has a small value; e.g. under vacuum (\( \varepsilon = \varepsilon_0 = 8.9 \times 10^{-12} \) F/m) the breakdown is about \( E = 10^6 \) V/m, yielding a maximum value of \( u_{\text{max}} = \frac{1}{2} (9 \times 10^{-12}) (10^6)^2 = 4.5 \) J/m\(^3\), or an electrostatic forces per unit area of 4.5 Pa, to be compared with typical chamber pressure of \( 10^7 \) Pa in chemical rockets.

ESA’s GOCE (Gravity Field and Steady-State Ocean Circulation Explorer) satellite (2009-2013) used a pair of ion thrusters (manufactured by Qinetic), power by 1.6 kW fixed solar panels, to accelerate \( 0.3 \times 10^{-6} \) kg/s of Xe ions to 40 km/s exit speed, producing a 0.01 N thrust to precisely compensate residual air drag at the very low Earth orbit of 260 km. The octagonal, 1100 kg satellite with a cross-sectional area of only 1 m\(^2\) was configured to keep aerodynamic drag and torque to an absolute minimum (without propulsion, deceleration would be \( 10^{-5} \) m/s\(^2\), i.e. 1 \( \mu \)g); it is symmetrical about its flight direction and two winglets provide additional aerodynamic stability.

Hall thrusters

A neutral gas (e.g. Xe) is injected into a Hall ionization chamber (through the anode, or close to it), and then electrostatically accelerated. The main difference with ion thrusters is that no grids are used to accelerate the ions with an E-field; instead, a strong radial magnetic field (B-field) is used to confine the electrons (strong enough to substantially deflect the low-mass electrons, but not the high-mass ions). The combination of the radial magnetic field and axial electric field cause the electrons to drift azimuthally, forming the Hall current from which the device gets its name. Hence, ions are accelerated electrostatically, but electrons are stopped magnetically, and the main contribution to thrust power is magnetic instead of electric. The energy-conversion efficiency is around 60 %.

Similar to electrostatic pressure, \( p = \frac{1}{2} \varepsilon |E|^2 \), there is a magnetic pressure, \( p = \frac{1}{2} \mu |B|^2 / \mu_0 \), where \( B \) is the field strength and \( \mu_0 = 1.256 \times 10^{-6} \) H/m is the permeability of vacuum. Without recourse to superconductive structures, \( B \) can be up to 0.1 Tesla (either using coils or permanent magnets), so that \( p_{\text{max}} = \frac{1}{2} (0.1)^2 / (10^6) = 5000 \) Pa (in practice just 10 to 100 times the value for ion thrusters, instead of 1000 times).

Electrospray thrusters (also named colloid thrusters) are based on ionic liquids, not on ionizing a neutral gas. They are special ion thrusters where heavy molecular ions (from a zero-vapour-pressure ionic liquid),
are extracted and accelerated by strong electric fields (the liquid is not pressurised, and flows by capillarity, without valves, i.e. like in inkjet printers). With only a small power (usually <10 W), they can provide specific impulse of 20–35 km/s, for attitude control and main propulsion in very small satellites like CubeSats (the standard size, 1U, is a cube of 1 L and has a mass <1 kg); the whole propulsion system may occupy 10–30 % of the size of a 1U CubeSat; e.g. <150 g of propellant would be required by a 1U CubeSat to reach Earth’s escape velocity from LEO and explore interplanetary space. Lisa Pathfinder technology-spacecraft, scheduled for 2015, will use US-built colloid thrusters and ESA-built gas thrusters.

Pulsed-plasma thrusters use high-voltage-arc pulses over a solid-propellant’s exposed surface (usually Teflon, PTFE), vaporizing it and creating an instant plasma; the resulting induced magnetic field accelerates the plasma. Because they operate in a pulsed mode, they don’t need continuous high power. Instead, they can gradually store electrical energy in a capacitor for release in high-power bursts. The propellant efficiency is high \( I_{sp} = 1000 \text{ s} \), but its energy-conversion efficiency (20 %) is lower than ion or Hall thrusters.

ROCKET PROPULSION MODELLING

Tsiolkovsky rocket equation

Consider the 1D motion of a rocket vehicle (in absence of weight and drag) at speed \( v \) (in a fix inertial reference frame), and ejecting propellant mass at a rate \( \dot{m}_p \) with an escape speed \( v_e \) relative to the body (i.e. at \( v_e - v \) in fix inertial axes). The momentum balance in the advance direction is \( \frac{d(mv)}{dt} + \dot{m}_p (v - v_e) = 0 \), which, with the mass balance \( \dot{m}_p = -\frac{dm}{dt} \) becomes:

\[
\frac{d(mv)}{dt} - \dot{m}_p (v - v_e) = m \frac{dv}{dt} + \frac{dm}{dt} v_e = 0 \quad \rightarrow \quad \frac{dv}{m} = -\frac{v_e}{v_e} = \Delta v = -v_e \ln \frac{m_{\text{final}}}{m_{\text{initial}}} \quad (3)
\]

known as the ideal rockets equation (or Tsiolkovsky equation, published in 1903). In summary, by ejecting mass, a rocket propels itself with ever-increasing speed \( v \) (with an acceleration \( a \) proportional to the exhausting mass-flow-rate and its exit speed, and inversely proportional to the rocket mass \( a = v_e \dot{m}_p/m \)), so that the finite change in rocket speed \( \Delta v \) after a period \( \Delta t \), is directly proportional to propellant exit speed (specific impulse, \( v_e \)) and to the logarithm of the initial-to-final mass ratio \( m_{\text{final}}/m_{\text{initial}} \) (the initial-minus-final rocket mass, \( m_{\text{final}} - m_{\text{initial}} \), is the propellant mass spent). Notice that the result in (3) was obtained with a 1D model, so that it can only be applied to periods with constant exhaust speed and little variation in the directions of thrust and velocity, what in orbital mechanics usually implies short thrust periods (impulse manoeuvres); for low thrust systems like electrical thrusters, this approximation is poor.

Exercise 2. What is the minimum fuel fraction for a single-stage rocket to reach low Earth orbit (LEO)?

Sol.: Basically one has to get orbital speed, some 7.8 km/s, so that Tsiolkovsky equation for a solid-fuel rocket with a typical specific impulse of \( I_{sp} = 250 \text{ s} \) (\( v_e = 2500 \text{ m/s} \)) yields \( m_{\text{final}}/m_{\text{initial}} = \exp(-\Delta v/v_e) = \exp(-7800/2500) = 0.044 \), i.e. more than 96% of the total lift-off mass must be fuel.

For comparison, the Shuttle had 2000 t at lift-off and 95 t in orbit (70 t the Orbiter and 25 t its payload), so that 95/2000 = 0.048; but the Shuttle used parallel stages of solid-fuel and cryogenic-
fuel rockets; for a LH2/LOX rocket with \( v_e = 4500 \text{ m/s}, \) \( m_{\text{fin}}/m_{\text{ini}} = \exp(-\Delta v/v_e) = \exp(-7800/4500) = 0.18. \) Notice that we have neglected differences in gravitational-potential energy and air drag, but the result is not out of scope. To be more precise, for a circular orbit at \( z = 300 \text{ km} \) (lower orbits decay in less than few weeks), orbital speed is \( v_{\text{orb}} = \sqrt{GM/(R+h)} = \sqrt{6.67 \times 10^{-11} \times 6.1 \times 10^2 / \left( (6.37 \times 10^6 + 300 \times 10^3) \right)} = 7730 \text{ m/s}. \) Earth's rotation at KSC, \( v_{\text{KSC}} = 2\pi R \cos \phi / T = 2\pi \times 6.37 \times 10^6 \times \cos(28.6^\circ) / 86164 = 410 \text{ m/s}, \) has not been accounted for, but it is a small help, if any. For a unit mass on the Earth surface, the rocket must supply not only the kinetic energy for a LEO orbit, \( \Delta E_k = (v_{\text{orb}}^2 - v_{\text{KSC}}^2)/2 = 30 \text{ MJ/kg}, \) but the potential energy to get there, too, \( \Delta E_p = -GM/(R+h) + GM/R = -63+59 = 4 \text{ MJ/kg}, \) and, on top of that, the lost work by air drag on the rocket must be added (what may require an additional 1 MJ/kg or so (the \( \Delta v = 1.5 \text{ km/s} \) assumed above). A total value of \( \Delta v = 9.4 \text{ km/s} \) is the current practical value to bring an object from ground to a 300 km LEO, what is equivalent to \( \frac{1}{2}v^2 = 44 \text{ MJ/kg}. \)

**Delta-v budget**

A 'delta-v budget' is a concise statement of the relative propellant mass required for the various propulsive tasks and orbital manoeuvres over one or more phases of a space mission; each \( \Delta v \) are computed by (3) under the assumptions there stated. Notice that, if rocket exhaust speed is the same, delta-v values are additive (mass ratios being multiplicative). Notice also that, in spite of non-dissipative orbital mechanics being reversible, rocket thrust always decreases propellant mass and hence delta-v must be considered as absolute (i.e. positive) values (e.g. if going from LEO to GEO requires a \( \Delta v = 4 \text{ km/s}, \) coming back from GEO to LEO would also require \( \Delta v = 4 \text{ km/s} \) (no refund).

The delta-v is not a measure of energy but of fuel-fraction needed, and, in spite of both being additive, delta-v is not conservative (depends on the path); e.g. to escape from Earth attraction starting at its surface, it is shown below that a single shot with \( \Delta v = 11.2 \text{ km/s} \) would do the job (leaving aside air-drag and Earth's rotation), whereas in a two-steps escape process, first going to a circular GEO (\( \Delta v = 12 \text{ km/s} \)), and then escaping (\( \Delta v = 1.3 \text{ km/s} \)) would need more propellant (12+1.3=13.3>11.2 \text{ km/s}). The reason is that the same delta-v yields different energy jumps: \( \Delta e = \frac{1}{2}(v+\Delta v)^2 - \frac{1}{2}v^2 = v\Delta v + \frac{1}{2}\Delta v^2, \) i.e. the energy jump not only depends on \( \Delta v \) but on the previous speed \( v \) (Oberth effect). The computations that follow hereafter illustrate the issue.

**NOTE.** All along this presentation on orbital mechanics, the Earth is assumed spherical, with a mean radius of \( R = 6371 \text{ km}, \) in spite of the fact that the mean Equatorial radius is 6378 km and the mean Polar radius 6357 km (i.e. a difference of 21 km); Earth's gravitational parameter is defined by \( \mu = GM = 6.674 \times 10^{11} \times 5.97 \times 10^{24} = 398 \times 10^{12} \text{ m}^3/\text{s}^2; \) Earth's rotation period (sidereal day) lasts 236 s less than the mean solar day, of 86400 s). Namely:

\[
\text{Earth's model: } R = 6371 \text{ km}, \mu = GM = 398 \times 10^{12} \text{ m}^3/\text{s}^2; \text{ } T_{\text{day}} = 86164 \text{ s} \quad (4)
\]

Table 1 presents some delta-v values. As said above, the rocket equation (3) shows that the payload-mass-fraction exponentially decreases with delta-v requirement, so that a large delta-v budget would benefit from staggered refuelling, as already practised on Polar exploration a century ago; intermediate propellant depots
should be established at Lagrangian points (in a halo orbit) to minimise orbit keeping, and in particular at the Earth-Moon-Lagrange-1 point (EML1), which is about 85 percent of the way to the Moon (about 58 000 from Moon's centre).

Table 1. Delta-v requirements assuming no air-drag, no planet-rotation, and short thrust period.

<table>
<thead>
<tr>
<th>Manoeuvre</th>
<th>∆v [m/s]</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>From ground to 100 km altitude</td>
<td>1400</td>
<td>See Suborbital flight, below.</td>
</tr>
<tr>
<td>From ground to escaping from Earth</td>
<td>11200</td>
<td>See Escape speed, below.</td>
</tr>
<tr>
<td>From ground to escaping from the Sun</td>
<td>42200</td>
<td></td>
</tr>
<tr>
<td>From rest to sea-level orbit</td>
<td>7930</td>
<td>Orbiting at zero altitude (H=0) with no drag.</td>
</tr>
<tr>
<td>-From sea-level orbit to 400 km LEO transfer</td>
<td>120</td>
<td>At H=400 km, ( v_{orbit}=7700 ) m/s.</td>
</tr>
<tr>
<td>-From LEO transfer to LEO circular</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>From rest to 400 km LEO</td>
<td>8168</td>
<td>8168=7930+120+118.</td>
</tr>
<tr>
<td>-From 400 km LEO to GEO transfer</td>
<td>2405</td>
<td></td>
</tr>
<tr>
<td>-From GEO transfer to GEO circular</td>
<td>1460</td>
<td></td>
</tr>
<tr>
<td>From 400 km LEO to GEO</td>
<td>3865</td>
<td>3865=2405+1460. At GEO ( v_{orbit}=3080 ) m/s.</td>
</tr>
<tr>
<td>From rest to GEO</td>
<td>12033</td>
<td>12033=8168+3865+930+120+118+2405+1460.</td>
</tr>
<tr>
<td>From GEO to escaping from Earth</td>
<td>1276</td>
<td>If added to ( v_{GEO}=3080 ) m/s, it gives the escape speed from GEO altitude: ( v=4360 ) m/s.</td>
</tr>
</tbody>
</table>

From ground to 100 km altitude
Consider the vertical launch of a rocket from sea level, to reach \( z=100 \) km altitude (see Sounding rockets, below). A simple estimation with planar Earth, and no air drag (the energy balance being \( \frac{1}{2}mv^2=mgz \)) yields \( \Delta v=\sqrt{2gz}=\sqrt{2\cdot9.8\cdot10^2}=1.4 \) km/s. In practice, some additional delta-v must be accounted for to compensate air drag.

Escape speed
For a body of mass \( m \) to escape from the gravitational attraction of a massive body \( M \), its kinetic energy \( (\frac{1}{2}mv^2) \) must overcome the gravitational potential energy \( -GMm/r \) (relative to zero potential at infinite distances). Notice that it is an energy balance in a ballistic shot (i.e. without further propulsion), the velocity direction being irrelevant (assuming the trajectory does not intersect the surface of the attracting body). The escape speed for a body at rest sitting at a radial distance \( r \) from the centre of mass is \( v=\sqrt{2\mu/r} \), where \( \mu=GM \) is the gravitational parameter (for Earth, \( \mu=GM=6.7\cdot10^{-11}\cdot6\cdot10^{24}=0.4\cdot10^{15} \) m\(^3\)/s\(^2\)).

The escape speed from Earth surface is \( v=\sqrt{2\mu/R}=\sqrt{2\cdot(400\cdot10^{12})/(6.37\cdot10^6)}=11.2 \) km/s. Notice that, as the Earth turns around the Sun, when a spacecraft escapes from Earth it is still attracted by the Sun (see A trip to Mars, below). To escape from the Solar System, a spacecraft at rest at a distance of 1 au (1 au=150\cdot10^9 km) from the Sun would require a kick of \( v=\sqrt{2\mu_s/R_{SE}}=\sqrt{2\cdot(6.67\cdot10^{-11}\cdot2\cdot10^{30})/(150\cdot10^9)}=42.2 \) km/s. Notice that, if a direct escape from Sun attraction were tried from Earth surface, a total of 11.2+42.2=53.4 km/s would be required.

If the escaping body has an initial speed, only the difference to the escape speed at that point should be added (if the push is along the moving direction); e.g. for an spacecraft at a 400 km altitude LEO with orbit speed of 7.7 km/s (see below) the additional \( \Delta v \) to escape Earth attraction is 10.9–7.7=3.2 km/s, since \( v=\sqrt{2\mu/(R+H)}=\sqrt{2\cdot(400\cdot10^{12})/(6.77\cdot10^6)}=10.87 \) km/s.

Spacecraft propulsion
To escape from Moon attraction, a spacecraft at its surface needs a push of \( v = \sqrt{\frac{2\mu_M}{r_M}} = \sqrt{2 \cdot \frac{6.67 \cdot 10^{-11} \cdot 73.5 \cdot 10^{21}}{(1.74 \cdot 10^6)}} = 2.37 \text{ km/s}; \) to escape from Mars, \( v = \sqrt{\frac{2GM_a}{r_a}} = \sqrt{2 \cdot 6.67 \cdot 10^{-11} \cdot 0.64 \cdot 10^{24} / (3.4 \cdot 10^6)} = 5.0 \text{ km/s}. \) Notice that for a spacecraft at mean Earth-Moon distance to escape from Earth's attraction, only a push of \( v = \sqrt{\frac{2GM_{EM}}{r_{EM}}} = \sqrt{2 \cdot 6.67 \cdot 10^{-11} \cdot 6 \cdot 10^{24} / (384 \cdot 10^6)} = 1.44 \text{ km/s} \) is needed. Notice that all the delta-v values computed above must be modified when air drag or planet rotation effects apply; e.g. to overcome air drag in Earth launches, a typical \( \Delta v = 1.5 \text{ km/s} \) must be added, and the entrainment by planet rotation is \( (2\pi/T_d)R \cos \phi \sin \alpha \), where \( T_d \) is the rotation period, \( R \) the radius, \( \phi \) the latitude of the launch site, and \( \alpha \) the azimuth (apparent launch inclination over the parallel, e.g. \( \alpha = 0 \) means due East, \( \alpha = 90^\circ \) due North); e.g. launching from the Equator due East on Earth already has \( v = (2\pi/T_d)R = (2\pi/86140) \cdot 6370 = 0.465 \text{ km/s} \) (the escape speed would be \( 11.2 - 0.5 = 10.7 \text{ km/s} \)), whereas for the Moon \( v = (2\pi/T_d)R = (2\pi/(2.36 \cdot 10^6)) \cdot 1740 = 0.005 \text{ km/s} \) (the escape speed would remain almost the same, 2.37 km/s).

From ground to LEO

Consider the specific-orbital-energy equation for elliptical orbits:

\[
\frac{v^2}{2} - \frac{\mu}{r} = \frac{\mu}{r_p + r_a} \quad \Rightarrow \quad v_c = \sqrt{\frac{\mu}{r}}, \quad v_p = \sqrt{\frac{2\mu}{r_p + r_a}}, \quad v_a = \sqrt{\frac{2\mu}{r_a + r_p + r_a}} \tag{5}
\]

where \( v \) and \( r \) are the spacecraft speed and distance to the mass centre of attraction, \( r_p \) and \( r_a \) are the periapsis and apoapsis of the orbit (\( v_p \) and \( v_a \) the corresponding speeds, with \( r_p v_p = r_a v_a \) by Kepler's 2nd law), and \( \mu \equiv GM \). The origin of potential energies is at \( r_a \to \infty \), and hence, at any point in any orbit, the potential energy (\( \mu/r \)) and total energy (right-hand-side of (5)) have negative values. As a corollary of (5), the escape speed from a point \( r \) is \( (r_a \to \infty) \quad v = \sqrt{2\mu/r} = \sqrt{2} v_c \), as made use above.

For a LEO of 400 km altitude (roughly applicable to the ISS), with \( r = R + H = 6370 + 400 = 6770 \text{ km} \), orbit speed is \( v = \sqrt{2\mu/r} = 7.7 \text{ km/s} \). The escape speed for a body at rest at 400 km altitude would be \( v = \sqrt{4\mu/r} = 10.9 \text{ km/s} \), and the three energy terms in the first of (5) are \( [v^2/2, -\mu/r, -\mu/(2r)] = [29.5, -59, -29.5] \text{ MJ/kg} \).

To reach an \( H \)-altitude LEO from ground, the best way (without air) would be to fire horizontally to get an orbit with an apogee radius \( r_a = R + H \) and a perigee \( r_p = R \); the required launching speed obtained from (5) with \( \mu = 0.4 \cdot 10^{15} \text{ m}^3/\text{s}^2 \) is \( v_p = 8.05 \text{ km/s} \) (more than the circular orbit speed at \( r = R = 6370 \text{ km}, 7.93 \text{ km/s} \), but less than the escape speed, 11.2 km/s). And, at the apogee of this transfer orbit, \( r_a = R + H = 6370 + 400 = 6770 \text{ km} \), to give another kick to circularise the orbit at 400 km altitude. As the circular speed is \( v = 7.69 \text{ km/s} \) (obtained from (5) with \( r = r_a = R + H = 6770 \text{ km} \)), and the speed at the apogee of the former transfer orbit is \( v = 7.57 \text{ km/s} \) (obtained from (5) with \( r = r_a = R + H \) and \( r_p = R \)), the delta-v here is \( \Delta v = 7.69 - 7.57 = 0.12 \text{ km/s} \), and the total delta-v (perigee plus apogee) \( \Delta v = 7.93 + 0.12 = 8.05 \text{ km/s} \). The energy analysis is as follows: when the body is at rest on ground \( (v = 0, r = R) \) the kinetic, potential, and total specific energies are \( [\varepsilon_{\text{kin}}, \varepsilon_{\text{pot}}, \varepsilon_{\text{tot}}] = [0, \varepsilon_{\text{kin}}, \varepsilon_{\text{pot}}] = \varepsilon_{\text{pot}} \).
−62.8, −32.4] MJ/kg; after the perigee kick (v=8.05 km/s, r=R) \([e_{\text{kin}},e_{\text{pot}},e_{\text{tot}}]=[v^2/2,−\mu/R,−\mu(2R+H)]=[32.4, −62.8, −30.5]\) MJ/kg; before the apogee kick (v=7.57 km/s, r=R+H) \([e_{\text{kin}},e_{\text{pot}},e_{\text{tot}}]=[v^2/2,−\mu(R+H),−\mu(2R+H)]=[28.7, −59.1, −30.5]\) MJ/kg; and after the apogee kick (v=7.93 km/s, r=R+H) \([e_{\text{kin}},e_{\text{pot}},e_{\text{tot}}]=[v^2/2,−\mu(R+H),−\mu(2R+H)]=[29.6, −59.1, −29.6]\) MJ/kg; the conservation of energy in this non-dissipative motion dictates that the initial energy at ground (−62.8 MJ/kg), plus the kinetic energy added in the perigee kick (8.05^2/2=32.3 MJ/kg), plus the kinetic energy added in the apogee kick (7.67^2/2−7.57^2/2=0.9 MJ/kg), equals the final total energy −29.6 MJ/kg.

As said above, besides this $$\Delta v$$ from ground to LEO, an additional $$\Delta v=1.5$$ km/s must be accounted for, to overcome atmospheric drag, perhaps reduced or increased by up to 0.46 km/s if the Earth's rotation entrainment speed ($$2 \pi R / T_d = (40 000 \text{ km})/(86140 \text{ s}) = 0.465 \text{ km/s}$$ at the Equator) is used for or against.

**LEO drag compensation**

A typical delta-v to compensate the thin atmosphere drag for a 400 km altitude is about $$\Delta v=30$$ m/s per year, corresponding to a residual air drag deceleration of 0.1 $$\mu g$$ all year around (2 m/s per year at 600 km altitude).

**Deorbit from LEO**

It depends on desired reentry angle (see Reentry, below); a typical value is $$\Delta v=0.15$$ km/s, to achieve a reentry angle of about 2°, but any value would do the job if no recovery is envisaged (even no impulse at all), since residual atmospheric drag will eventually slow down any orbiting object (e.g. an object at 300 km altitude in circular orbit would take several months, up to a year if massive, to be swallowed; see Deorbiting, below).

From LEO to GEO

The geostationary Earth orbit (GEO) is equatorial, due East, and with a sidereal-day period, $$T_d=86164$$ s, so that its radius is (from $$F=G M m / r^2 = m \alpha r = m (2 \pi / T_d)^2 r$$), $$r=42 000 \text{ km}$$ (H=35 800 km altitude), and its orbital speed is $$v=\sqrt{GM/r} = \sqrt{0.4 \cdot 10^5 / (42 \cdot 10^6)} = 0.3 \text{ km/s}$$.

To go from LEO (assumed equatorial to be coplanar, due East to minimise $$\Delta v$$, and of H=400 km altitude) to GEO, the best approach with short-period boosts is a Hohmann transfer, with a perigee kick at LEO and an apogee kick at GEO (similarly as the transfer from ground to LEO explained above). The four stages are:

- At a $$H=400$$ km ($$v=7.7 \text{ km/s}$$, $$r=R+H=r_{\text{LEO}}$$) \([e_{\text{kin}},e_{\text{pot}},e_{\text{tot}}]=[v^2/2,−\mu/r_{\text{LEO}},−\mu(2r_{\text{LEO}})]=[29.6, −59.1, −29.6]\) MJ/kg.
- After the perigee kick of $$\Delta v=2.40$$ km/s ($$v=10.1 \text{ km/s}$$, $$r=r_{\text{LEO}}$$) \([e_{\text{kin}},e_{\text{pot}},e_{\text{tot}}]=[v^2/2,−\mu/r_{\text{LEO}},−\mu(r_{\text{LEO}}+r_{\text{GEO}})]=[50.9, −59.1, −8.2]\) MJ/kg. This boot adds $$\Delta e_{\text{kin}}=\Delta (v^2/2)=21.4$$ MJ/kg.
- Before the apogee kick ($$v=1.62 \text{ km/s}$$, $$r=r_{\text{GEO}}$$) \([e_{\text{kin}},e_{\text{pot}},e_{\text{tot}}]=[v^2/2,−\mu/r_{\text{GEO}},−\mu(r_{\text{GEO}}+r_{\text{GEO}})]=[13.1, −9.5, −8.2]\) MJ/kg.
- After the apogee kick of $$\Delta v=1.46$$ km/s ($$v=3.1 \text{ km/s}$$, $$r=r_{\text{GEO}}$$) \([e_{\text{kin}},e_{\text{pot}},e_{\text{tot}}]=[v^2/2,−\mu/r_{\text{GEO}},−\mu(2r_{\text{GEO}})]=[47.5, −9.5, −4.75]\) MJ/kg. This boot adds $$\Delta e_{\text{kin}}=\Delta (v^2/2)=3.4$$ MJ/kg.
The energy increment from LEO to GEO is then \(-4.75 - (-29.6) = 24.85\) MJ/kg, which coincides with the sum of kinetic energy increments at perigee (21.4 MJ/kg) and at apogee (3.4 MJ/kg).

From ground to GEO (without going through LEO)
To reach GEO directly from ground the best way (without air) would be to fire horizontally to get an orbit with an apogee radius \(r_a = r_{GEO} = 42000 \text{ km}\) and a perigee \(r_p = R = 6370 \text{ km}\); the required launching speed obtained from (5) is \(v_p = 10.45 \text{ km/s}\) (more than the circular orbit speed at \(r = R\), which is 7.93 km/s, but less than the escape speed, 11.2 km/s). And, at the apogee of this transfer orbit give, another kick to circularise the orbit at GEO. The four stages would be:

- At rest on the ground \((H=0, v=0, r=R)\) \([e_{\text{kin}}, e_{\text{pot}}, e_{\text{tot}}] = [0, -62.8, -62.8] \text{ MJ/kg}\).
- After the perigee kick of \(\Delta v = 10.1 \text{ km/s}\) \((v=10.1 \text{ km/s}, r=R)\) \([e_{\text{kin}}, e_{\text{pot}}, e_{\text{tot}}] = [v^2/2, -\mu/R, -\mu/(R+r_{GEO})] = [54.58, -62.83, -8.24] \text{ MJ/kg}\). This boot adds \(\Delta e_{\text{kin}} = \Delta (v^2/2) = 54.6 \text{ MJ/kg}\).
- Before the apogee kick \((v=1.58 \text{ km/s}, r=r_{GEO})\) \([e_{\text{kin}}, e_{\text{pot}}, e_{\text{tot}}] = [v^2/2, -\mu/r_{GEO}, -\mu/(R+r_{GEO})] = [1.25, -9.49, -8.24] \text{ MJ/kg}\).
- After the apogee kick of \(\Delta v = 1.50 \text{ km/s}\) \((v=3.1 \text{ km/s}, r=r_{GEO})\) \([e_{\text{kin}}, e_{\text{pot}}, e_{\text{tot}}] = [v^2/2, -\mu/r_{GEO}, -\mu/(2r_{GEO})] = [47.5, -9.5, -4.75] \text{ MJ/kg}\). This boot adds \(\Delta e_{\text{kin}} = \Delta (v^2/2) = 3.5 \text{ MJ/kg}\).

The energy increment from ground to GEO is \(-4.75 - (-62.8) = 58.05 \text{ MJ/kg}\), which coincides with the sum of kinetic energy increments at perigee (54.6 MJ/kg) and at apogee (3.5 MJ/kg). Notice that the global delta-v from ground to GEO without parking at LEO, \(\Delta v = 10.1 + 1.5 = 11.6 \text{ km/s}\), is slightly lower than if going through LEO \((7.93 + 0.12 + 20.40 + 1.46 = 11.9 \text{ km/s})\). As said above, besides this delta-v budget, an additional \(\Delta v = 1.5 \text{ km/s}\) must be accounted for in launchers to overcome atmospheric drag, reduced by up to 0.465 km/s if the Earth's rotation entrainment speed \(2\pi R/T_{\text{day}} \cos \phi = (40000 \text{ km})/(86140 \text{ s}) \cos \phi\), \(\phi\) being the launch-site latitude, is advantageously used.

Station keeping at GEO
A typical delta-v of about 50 m/s per year should be envisaged, not to compensate atmosphere drag, which is irrelevant, but to compensate for North-South disturbances caused by solar and lunar attraction, for East-West disturbances caused by non-uniform geoid attraction, and by solar radiation pressure.

From GEO to graveyard orbit
It would be too expensive to deorbit from GEO; essentially a \(\Delta v\) equal to the 1.5 km/s computed above for the apogee kick in the LEO to GEO transfer. To free these valuable vantage points at the end of the useful life of spacecraft (usually dictated by the lack of propellant to keep the station), GEO satellites are sent to a higher orbit about 300 km above GEO (named disposal or graveyard orbit), spending only about \(\Delta v = 11 \text{ m/s}\) (the same amount of fuel that a satellite needs for approximately three months of station-keeping). While most satellite operators try to perform such a manoeuvre at the end of the operational life (and there are several GEO satellites going out of service every year), only one-third succeed in doing so, and there are proposals to send a dedicated spacecraft to bring dead GEO satellites to the graveyard, either by pushing them, or perhaps by refilling some propellant if allowed (what might be used to extend their operational life, if not obsolete).
For medium Earth orbits (MEO) like the semi-synchronous orbit (at around 14 000 km altitude), mainly used by navigation satellites (GPS and Galileo), the graveyard orbit is some 600 km above the operational MEO. After reaching the graveyard orbit, the propellant remaining in the satellite is dumped (and batteries discharged) to avoid a possible explosion in the event of a collision, what would worsen the debris problem, throwing pieces into the operational orbits.

From Moon's surface to low lunar orbit (LLO)
Instead of going through the details as above, a quick estimate can be done knowing that Moon's gravity is 1/6 of Earth's, and Moon's radius 1/4 of Earth's radius, approximately, and hence \( \Delta v = \sqrt{g R / 6 / 4} \) = (7.9 km/s) / \( \sqrt{24} \) = 1.6 km/s. There is no air drag here, and the entrainment speed due to Moon's rotation is negligible (4.6 m/s at its Equator).

Gravity assist
The gravitational attraction of a planet or moon with a linear speed \( v_{\text{planet}} \) in an inertial frame at speed (relatively to the Sun, approximately), may be used to add or subtract some \( \Delta v \) to the absolute speed of a spacecraft, with the limit \( \Delta v = 2v_{\text{planet}} \). For the Earth, \( v_{\text{planet}} = 2\pi R_{\text{SE}} / T_{\text{year}} = 30 \) km/s.

Notice that, in a launch to a planet, because the relative positions of planets change with time, the delta-\( v \) budget changes with launch dates.

Rocket exit speed
The exhaust gas speed in rockets must be very large (thrust \( F = \dot{m}_p v_e + A_e (p_e - p_0) \approx \dot{m}_p v_e = \rho_e v_e^2 A_e \) is proportional to \( v_e^2 \)); that is why there is such a strong roaring at lift-off (some 190 dB in Space Shuttle launches), caused by the shock waves created by the supersonic exhaust jet when mixing with ambient air.

Exit speed in a rocket is limited by available energy in the propellants (energy by unit mass, i.e. specific energy), or by available power from outside (e.g. if solar powered); in the latter case the limit is set by \( \dot{W} = \frac{1}{2} \dot{m}_p v_e^2 \), although, if external energy is captured and stored aboard for short-time usage, then the specific-energy-storage limit applies.

The specific energy storage in cold-gas propellants is the internal thermal energy of the gas in the chamber relative to the cold limit of isentropic expansion to vacuum; with the perfect gas model \( E / m = c_p T \), so that exit speed is limited (from \( E = \frac{1}{2} m v^2 \)) to \( v_e = \sqrt{2 c_p T} \), or, if exit pressure is not zero but \( p_e \), and chamber pressure is \( p_c \), then:

\[
v_e = \sqrt{2 c_p (T_e - T_c)} = \sqrt{2 c_p T_c \left[ 1 - \left( \frac{p_e}{p_c} \right)^{\frac{T_c}{T_e}} \right]}
\]

For instance, maximum exit speed for a N\(_2\) thruster in vacuum is \( v_e = \sqrt{2 c_p T} = \sqrt{2 \cdot 1000 \cdot 300} = 770 \) m/s (\( I_{sp} = 77 \) s). However, using a resistojet where nitrogen is heated to, say, 1200 K (what requires a heating of \( c_p \Delta T = 1000 \cdot (1200 - 300) = 0.9 \) MJ/kg), rises the vacuum exit speed to \( v_e = \sqrt{2 c_p T} = \sqrt{2 \cdot 1000 \cdot 1200} = 1500 \) m/s.
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m/s ($I_{sp}=150$ s), even without accounting for the increased $c_p$ with temperature. To see the effect of back-pressure, for $p_e=100$ kPa (we need a higher chamber pressure, e.g. $p_c=1$ MPa), the 300 K nitrogen would exit at $v_e = \sqrt{2c_pT} = \sqrt{2\cdot1000\cdot300\left[1-(1/10)^{0.4/1.4}\right]}=540$ m/s, and the 1200 K nitrogen would exit at 1080 m/s.

The specific energy storage in chemically-reactive propellants is the lower heating value of propellants (water formed will be in gas state at the exhaust $T$p-conditions). Mind the difference with the LHV of fuels; e.g. the LHV of hydrogen is 242 kJ/mol=121 MJ/kg (the burning of 1 kg of H$_2$ with O$_2$ yields 121 MJ, if enough oxygen is available), but, by mass of products, the heating value at the stoichiometric ratio is 121·0.002/0.018=13 MJ/kg (i.e. every 1 kg of H$_2$O formed gets 13 MJ). Exit speed is limited (from $E=\frac{1}{2}m v^2$) to $v_e = \sqrt{2E/m}$; e.g. maximum exit speed for H$_2$/O$_2$ burning is $v_e = \sqrt{2E/m} = \sqrt{2\cdot13\cdot10^6} = 5200$ m/s, although exhaust pressure, dissociation, and the use of non-stoichiometric mixtures, reduce this value to about 4500 m/s. In the case of a solid propellant like APCP, with a lower heating value of about 4 MJ/kg, exit speed is around $v_e = \sqrt{2E/m} = \sqrt{2\cdot4\cdot10^6} = 2800$ m/s.

The equilibrium combustion package CEA (Chemical Equilibrium with Applications, from NASA) can also be used to perform equilibrium chemistry calculations and has a capability similar to STANJAN-1970s but with a much wider range of chemicals.

Exercise 3. A small 50 kg satellite is to be equipped with a resistojet thruster using butane stored as a liquefied gas at 300 K, expected to give a thrust of 50 mN and a total delta-v of 10 m/s. Find:

a) Maximum exit velocity of the gas without adding energy in the resistors.

b) Minimum amount of propellant needed without adding energy in the resistors.

c) Propellant flow rate needed.

d) Assuming that an electrical resistance of 20 W is allowed to heat the gas at a steady state, find the gas temperature before expansion.

e) Change in exit speed, amount of propellant and thrusting time, when working as a resistojet

Sol.: a) Assuming that only gas is allowed to escape from the reservoir (e.g. by using capillary liquid traps inside the tank, except near the mouth), and that the nozzle allows a gas expansion to almost vacuum exit pressure, exit speed is $v_e = \sqrt{2c_pT} = \sqrt{2\cdot1800\cdot300} = 1040$ m/s ($I_{sp}=v_e/g=106$ s), where $c_p=1800$ J/(kg·K) is the gas thermal capacity at 300 K and its equilibrium vapour pressure, $p_v=258$ kPa.

b) With that exit speed, the total delta-v desired, and Tsiolkovski rocket equation (3), which can be linearized for small propellant masses, one gets $m_p=m\cdot \Delta v/v_e=50\cdot10/1040=0.48$ kg. With a density of $\rho=570$ kg/m$^3$ for liquid butane at 300 K, the tank must have a capacity larger than 0.48/0.570=0.84 litres.

c) For a thrust of 50 mN, the propellant mass flow rate must be $\dot{m}_p=F/v_e=50\cdot10^{-3}/1040=48\cdot10^{-6}$ kg/s. At that rate, the 0.48 kg of minimum propellant load would last 10 000 s (2.8 h).

d) If the 48·10$^{-6}$ kg/s of butane-gas enters the chamber at 300 K, and gets the full 20 W (e.g. the gas enters through a coiled tube wrapped around the exterior of a cylindrical solid resistor), the output temperature at the chamber would be $T_e = T_o + W_{ok}/(m_p c_p) = 300+20/(48\cdot10^{-6}\cdot1800)=531$ K, or better 300+20/(48·10$^{-6}$·2200)=490 K, if a more appropriate $c_p$-value at higher temperatures is used.

By the way, notice that the liquid reservoir must be let to recover its warmth, since the escape of the
48·10⁻⁶ kg/s of butane-gas must be compensated by vaporizing the same amount of liquid, absorbing 
\[ \dot{Q}_{\text{tank}} = \dot{m}_L h_{lv} = 48\cdot10^{-6}\cdot360\cdot10^3 = 17 \text{ W} \] (the boiling enthalpy at 300 K is 360 kJ/kg).

e) With a gas temperature in the chamber of 490 K, and using \( c_p = 2200 \text{ J/(kg·K) as mean value}, \) the exit speed may reach
\[ v_e = \sqrt{2c_p T} = \sqrt{2\cdot2200\cdot490} = 1470 \text{ m/s (I}_{\text{sp}} = 150 \text{ s}); \] the amount of propellant needed is now
\[ m_p = \dot{m}_p \Delta v/v_e = 50\cdot10/1470 = 0.34 \text{ kg (0.60 L), the flow rate } \dot{m}_p = F/v_e = 50\cdot10^{-3}/1470 = 34\cdot10^{-6} \text{ kg/s}, \] and hence the maximum thrusting time remains the same, 10 000 s (2.8 h).

**Chamber pressure equation**

We have seen that, in chemical rockets, chamber temperature is almost fixed by adiabatic combustion, depending basically on the type of propellants and their mixing ratio, and being almost independent on pressure, size, or total flow rate (temperature is a maximum near stoichiometric oxidiser/fuel ratio, and drops in both rich and lean mixtures). Chamber pressure, however, is controlled by a mass balance between input and exit flows, \( \dot{m}_\text{in} - \dot{m}_\text{out} = \dot{m}_\text{in} - \dot{m}_\text{exit}, \) where the input flow \( \dot{m}_\text{in} \) is fed through injectors in liquid and gas rockets, and by surface recession of the burning propellant in solid rockets (where the combustion chamber and the propellant tanks are the same). Chamber pressure has little effects on combustion efficiency or in specific impulse; its main effect being on the exit flow-rate \( \dot{m}_\text{out}, \) which is limited by flow choking at the nozzle throat (depending on chamber pressure).

In liquid rockets the input flow is imposed by turbopumps, with little influence of chamber pressure, but in solid rockets there is an important pressure effect (and recall that, once ignited, the propellant burns until exhaustion): the solid burning rate or surface receding speed, \( v_r, \) is modelled by the burning-rate law (also known as Saint-Robert's law, or Vieille's Law, 1893):

\[ v_r = \frac{K}{T_c} p_c^n \]  \hspace{1cm} (7)

where \( K \) is a constant (depending on propellant composition), \( T_c \) and \( p_c \) are the temperature and pressure in the combustion chamber, and \( n \) is a lower-than-unity exponent, \( 0.4 < n < 0.7, \) because, in order not to run away (i.e. to be sub-critically pressure-sensitive), the higher the pressure the lower the \( \frac{dv_r}{dp} \) must be (assuming constant surface burning area). Sometimes the temperature difference between chamber-gas and solid-propellant is used in (7), i.e. putting \( T_c - T_p \) instead of \( T_c, \) but the difference is negligible. The burning rate of AP grows almost linearly from 5 mm/s at 4 MPa to 11 mm/s at 10 MPa. The generated mass-flow-rate is:

\[ \dot{m}_\text{gen} = \rho_s A_1 v_r = \rho_s A_1 \frac{K p_c^n}{T_c} \]  \hspace{1cm} (8)

with \( A_1 \) the receding burning surface area, and \( K \) as defined in (7). The exit flow-rate is computed with nozzle-flow equations (see Nozzles), although subsonic flow should be considered during the short start-up and tail-off periods, so that:
if supersonic, \( \dot{m}_{\text{exit}} = \rho^* v^* A^* = \frac{p^*}{RT^*} \sqrt{\gamma RT^*} A^* = \frac{\gamma p^* A^*}{\sqrt{\gamma RT^*}} \left( \frac{2}{\gamma+1} \right)^{\gamma+1} \)

or in any case, \( \dot{m}_{\text{exit}} = \rho_c v_c A_e = \frac{\gamma p_c A_e}{\sqrt{\gamma RT_e}} M_e \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right) \)

The chamber pressure equation is then derived from the mass balance of gases as follows:

\[
\frac{d\dot{m}}{dt} = \dot{m}_{\text{gen}} - \dot{m}_{\text{exit}} = \rho_s A_t V = \frac{d(p_c V)}{RT_e dt} = \frac{V}{RT_e} \frac{dp_c}{dt} = \frac{V}{RT_e} \frac{dp_c}{dt} + \frac{dp_c}{RT_e} A_e v_e
\]

\[
\begin{align*}
\dot{m}_{\text{gen}} &= \rho_s A_t v_t = \rho_s A_t \frac{K p_e^n}{T_e} \\
\dot{m}_{\text{exit}} &= \rho_c v_c A_e = \frac{\gamma p_c A_e}{\sqrt{\gamma RT_e}} M_e \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right)^{2-\gamma} \text{ if choked} \\
&= \frac{\gamma p_c A_e}{\sqrt{\gamma RT_e}} \left( \frac{2}{\gamma+1} \right)^{\gamma+1} M_e \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right)^{2-\gamma} \end{align*}
\]

Neglecting gas density in the chamber \((p_c/(RT_e) \approx 10^7/(500 \cdot 3000) \approx 10 \text{ kg/m}^3)\) against solid propellant density \((\rho_s \approx 2000 \text{ kg/m}^3)\), dividing by \(V/(RT_e)\), and introducing function \(f_e\) of \(\gamma\) and \(M_e\) (considering the equation relating choked nozzle area \(A^*\) with nozzle exit area \(A_e\)), one gets the general chamber pressure equation for solid rockets:

\[
\frac{V}{RT_e} \frac{dp_e}{dt} = \dot{m}_{\text{gen}} - \dot{m}_{\text{exit}} = \rho_s A_t \frac{K p_e^n}{T_e} - \frac{\gamma p_c A_e}{\sqrt{\gamma RT_e}} M_e \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right)^{2-\gamma} \]

\[
\rightarrow \frac{dp_e}{dt} = \rho_s A_t R K p_e^n - \frac{\gamma RT_e p_c A_e f_e}{V}
\]

with \(f_e \equiv M_e \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right)^{2-\gamma} \text{ if choked} = A_e^* \left( \frac{2}{\gamma+1} \right)^{\gamma+1} \left( \frac{\gamma + 1}{2} \right)^{2\gamma+1} \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right)^{2-\gamma}\)

Notice that, although gas temperature may go over 3000 K, temperature at the burning solid surface hardly reaches 1000 K. During the transient start-up phase, gas generation outgrows exit flow, and chamber pressure increases rapidly to a quasi-steady state where exit mass of choked flow at the nozzle balances gas-generation flow. The steady-state chamber pressure is:

\[
p_e = \left( \frac{\rho_s A_t R K}{\sqrt{\gamma RT_e A_e f_e}} \right)^{1/n} = K^* \left( \frac{A_t}{A_e} \right)^{1/n}
\]

where \(K^* \equiv \left( \frac{\rho_s R K A_e}{\sqrt{\gamma RT_e A_e f_e}} \right)^{1/n} = \left( \frac{\rho_s R K}{\sqrt{\gamma RT_e}} \left( \frac{\gamma + 1}{2} \right)^{\gamma+1} \right)^{1/n}\)

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showing that, for a given propellant properties (solid density \( \rho_s \), gas constant \( R=R_u/M \), gas thermal capacity ratio \( \gamma=c_p/c_v \), adiabatic combustion temperature \( T_c \), and corresponding sound speed in the chamber \( c=\sqrt{\gamma RT_c} \)), chamber pressure grows to a maximum value that grows with the burning-to-throat area ratio \( A_r/A^* \), \( K' \) being a constant for a given propellant.

For a constant environmental pressure, after ignition, chamber pressure \( p_c \) first grows quadratically with time during subsonic exit (until the throat becomes choked), then \( p_c \) grows almost linearly with time while a normal chock wave moves from throat to exit section (if external pressure allows so, i.e. in over-expanded flows), and finally \( p_c \) slowly grows with time, exponentially approaching the steady value while the shock waves at the exit get oblique and less strong, or even expansion waves form if external pressure allows the flow to become under-expanded. The sequence of events is just in opposite order for pressure blow-down after combustion ends.

The tangential speed of burnt gases in the chamber (linearly increasing towards the exit) may give rise to erosive burning, i.e. an increase in the propellant burning rate (surface receding) as a result of the high velocity of combustion gases flowing over the surface.

**Rocket cooling**

Temperature in the rocket chamber, \( T_c \), is what yields specific thrust, since, in the limit, \( \frac{F}{m_p} = v_e = \sqrt{2c_pT_c} \). Except in cold-gas rockets, the high temperatures reached in the combustion chamber (or the plasma-generation chamber) and along the nozzle, make rocket cooling a critical technology, ranking in efficiency from transpiration cooling to radiative cooling. The usual cooling technique are:

- Injection of cold gases at the internal boundary layer, either at all points in the wall (transpiration cooling), or at some particular locations (at the injectors, mid-way along the combustion chamber, near the nozzle throat...). The cold gas should be generated on spot, usually by proper very-rich or very-lean combustion. The cooler gas layer is used on both liquid and solid rockets (Fig. 3).
- Forced flow along channels built in the wall (or ducts wrapping around), usually forcing the liquid fuel in counter-flow. In LH2/LOX engines, hydrogen enters as a compressed cryogenic liquid and becomes supercritical as it heats up from some 25 K to some 250 K before being injected.
- Ablative cooling. Only valid on one-shot low-duration rockets. In solid-fuel rockets, it is common to have some ablative material at places not covered by the grain (near the nozzle entrance and at the back of the chamber). Single-use liquid rockets may have the combustion chamber coated with several millimetre-thick, poorly conducting, silica-reinforced epoxy or phenolic resins.
- Passive cooling by radiation. Radiative cooling may work if the envelop surface is left to heat up to >2000 K, what requires refractory materials, either pure metals (wolfram \( T_m=3695 \) K, rhenium, tantalum), ceramic (carbon, silicon carbide, silicon nitride, boron nitride...), or ceramic-metallic composites. Both the Apollo Lunar Module (LM) and the Command/Service Module (CSM) had their hydrazine rocket nozzles made of refractory niobium alloys. Aerojet Rocketdyne is developing a radiation-cooled LOX/LNG-rocket of \( F=45 \text{ kN} \) and \( I_{sp}=350 \text{ s} \) for ascent from the lunar surface.
- No cooling. Relying on thermal inertia is only valid for small rockets or very short-duration pulses.
Fig. 3. Using a colder boundary-layer gas flow inside rocket walls: a) As used in liquid propellants, where both the nozzle and the combustion chamber are so cooled, besides the external counter-flow liquid cooling). b) As used in solid propellant rockets.

For instance, in the Space Shuttle main engine (SME), the main combustion chamber and the nozzle walls are made of Inconel-718 with a maximum service temperature of 950 K, while the burnt gases inside are at 3250 K, but this is cooled by supercritical hydrogen outside, with an average heat flux of 150 MW/m² (some 100 MW/m² at the chamber wall and more than double near the throat). However large this cooling rate seems to be, it is a mere 1.3% of the exiting enthalpy flow, which, for a 520 kg/s flow of gases with an average thermal capacity of 3500 J/(kg·K) at 3250 K (with a reference at 0 K), is \( \dot{H} = \dot{m}c_pT_0 = 520 \cdot 3500 \cdot 3250 = 5.9 \text{ GW} \). Effectively, cooling area is about \( A_{\text{wall}} = 0.5 \text{ m}^2 \), and thus \( \dot{Q} = \dot{q}A = 150 \cdot 10^6 \cdot 0.5 = 75 \text{ MW} \) and \( \dot{Q}/\dot{H} = 75/5900 = 1.3\% \).

**SUB-ORBITAL SPACEFLIGHT**

The International Astronautical Federation fixed 100 km altitude as the boundary between Earth's atmosphere and outer space, and named it the Kármán line; Theodore von Kármán had deduced in the 1950s that a vehicle would have to fly faster than orbital velocity to have sufficient aerodynamic lift from the air to stay aloft at that altitude.

There is a large range of altitudes unsuitable to 'level flight', say from 25 km to 250 km altitude; i.e. from the highest flight levels of supersonic aircraft, to the lower orbiting satellites (the lowest to date, GOCE, already needed some permanent propulsion to compensate residual air drag at 275 km altitude). Aerostats may raise the lower bound, since research balloons may reach almost 50 km altitude, but the only means to visit the 50..250 km altitude layer is by a quick flyby on a rocket, either on a up-and-down return (sub-orbital flight), or in a one-way to orbit or back journey. When a vehicle goes over 100 km altitude it is named a spacecraft (and people within are said astronauts).

Astronautics started in 1957 with the first orbital flight achieved by USSR-Sputnik, although the first sub-orbital flight was achieved in 1944 by German V-2 rocket, developed following von Braun's thesis "Construction, Theoretical, and Experimental Solution to the Problem of the Liquid Propellant Rocket"
(1934), using a bipropellant combination of 4910 kg of LOX, and 3810 kg of 75% ethanol / 25% water; fuel and oxidiser were introduced in the combustion chamber using turbopumps driven by the steam generated by concentrated hydrogen peroxide with sodium permanganate catalyst. The rocket reached 200 km altitude on tests, but altitude was limited to about 90 km when used in warfare operations to reach a 320 km horizontal range, in spite of the fact that a simple ballistic parabolic flight needs less propulsion. Maximum speed was 1.6 km/s. An ICBM typically has 3..5 minutes of propulsion, some 25 min of ballistic flight with apogee at 1200 km and ground displacement of around 20 000 km, and a quick ballistic reentry lasting about 2 min.

Suborbital flights were performed in preparation of orbital flights, and of human spaceflight, first achieved on 12 April 1961 by the Soviet Union with Yuri Gagarin in a single-orbit flight, and closely followed by the USA with the suborbital flight of Alan Shepard on 5 May 1961. In the latter case (Project Mercury), a Redstone rocket boosted during 150 s up to a height of 59 km, followed by a ballistic flight up to 190 km altitude 5 min after lift-off (here, the spacecraft's retrorockets were fire-tested for future orbit insertion), and the capsule landed at sea 485 km downrange, 15 min after lift-off.

Sub-orbital flights have remained of interest for intercontinental missiles, and for scientific research flights (first to sample the ionosphere layers, and later as a microgravity carrier); and is gaining commercial interest for space tourism (to see the Earth below and feel weightless); SpaceShipTwo will offer suborbital flights to 110 km altitude on a 6 passenger and 2 crew spacecraft of 9700 kg, powered by liquid/solid hybrid rocket engines, launched from a jet-powered mother aircraft.

Although these altitudes may be reached with a totally ballistic flight (a 180 kg bullet was shot with a gun at 3.6 km/s to 180 km altitude), sub-orbital flight is based on rocket engines. Rescue rockets used as safety devices in crewed launchers do not reach such high altitudes.

**Sounding rockets**

Sounding rockets are unmanned instrument-carriers for scientific research in sub-orbital flight, either to measure environmental variables (in the 50..300 km altitude range, because further upwards orbiting satellites are much better, and, below 50 km, balloons and aircraft are the best), or to provide a medium-term microgravity platform (in the 20 s to 20 minutes range, since shorter times are best obtained in drop towers and aircraft in parabolic flight, and longer periods require an orbiting spacecraft).

The flight path is usually almost vertical (5..10° zenith slant), with the craft landing relatively close to its take-off site (say within a 100..200 km radial distance). After the rocket propulsion phase (a solid propellant rocket burning for 5..100 s), the sounding rocket performs a ballistic flight following an elliptic orbit with apogee in the 100..1500 km range (the perigee would be below Earth surface), and re-enters the atmosphere at hypersonic speed, decelerating by air drag, and opening a parachute at some 3 km altitude to protect the payload landing. Two types of reentry attitude can be considered:

- Nose-first reentry configuration, as used in missiles, to minimise air drag and range uncertainty. It requires a 180° rotation manoeuvre (usually performed near the top of the path, after de-spinning in case the ascent phase is spin-stabilised.)
• Tail-first reentry configuration, which is the simpler solution, since the rocket is usually spin-stabilised during ascent. Besides, it allows nose parachutes (drogue and main) to be deployed easier (see Fig. 4).

ESA started using sounding rockets for microgravity (µg) research in 1982 with TEXUS, a European/German programme, offering 6 minutes of µg time to a payload of 400 kg (shared among several experimental modules), with apogee at 260 km. Launches are conducted from Esrange in Kiruna (Sweden), using a British Skylark rocket (from TX-1 in 1977 to TX-41 in 2004 when the Skylark program was ended; from that on, Orion rockets are used). Skylark had a Goldflinch first stage (2 m long) for take-off (TO), burning for 3.5 s and released 2 s later, and a Raven second stage (5 m long) burning from TO+6.5 s after lift-off to TO+45 s, a total of 840 kg of APCP propellant (ammonium perchlorate, polyisobutylene binder and aluminium powder), with a thrust of 44 kN at sea level. The µg period was almost 6 minutes (from TO+75 s to TO+400 s), with a deceleration during reentry below 20 g (in any axis); landing velocity was about 8 m/s, and, after the impact, the payload remained exposed for one or two hours to snow and cold air down to −30 ºC; during ascent, the skin attained more than 100 ºC at about TO+50 s, with a maximum of over 200 ºC during reentry; see the flight timeline in Fig. 4.
MAXUS is a larger sounding rocket used for similar purposes: 15 minutes of microgravity time, 800 kg payload, 700 km apogee, propelled by a Castor 4B rocket of 500 kN maximum thrust with 10 000 kg of solid propellant burning during 63 s to impose a maximum speed of 3.5 km/s, and so on. MiniTEXUS had an apogee of 140 km. REXUS (Rocket-borne EXperiments for University Students) is a much smaller sounding rocket endeavour, based on US-Orion rocket with typical apogee around 85 km (below the 100 km Kármán line), but some improved versions can go higher up to sub-orbital flight. To avoid pressure build-up within the payload and to avoid gas-flow between modules, it is sometimes necessary to have venting holes in experiment modules, but reaction forces from exhaust openings must be minimised (e.g. by using at least two openings located symmetrically on the module). If some payload needs direct exposure to the space environment, a payload fairing may be jettisoned when air drag is already negligible (say above \( z > 50 \) km, where \( p < 100 \) Pa, about TO+60 s). Soon after, a possible yo-yo de-spin system may be used to cancel the rocket rolling often used for trajectory stabilisation, and the rocket engine detached from the payload (Fig. 4). Rexus leaves the launcher rail with a 5º tilt to the vertical (strictly-vertical launch is always...
avoided for safety reasons), and attains a 4 rps spin after burnout (at TO+26 s); Fig. 5 presents the nominal trajectory data. It can be noticed that the $z(t)$ curve is concave while powered (positive acceleration), and convex during coasting (negative acceleration), and that the speed is not zero at the apogee (but the vertical component is zero); apogee is at TO+140 s, and parachute deploys at TO+250 s.

![Fig. 5. Rexus nominal flight trajectory.](image)

Leaving aside (for the moment) air-drag, the most efficient propulsion to reach a given altitude is not a rocket but a cannon ball, where all the propellant and engine is left on ground (any other means of ground acceleration, e.g. along a rail, would be also much better than a rocket). But the initial acceleration and speed would be very high, and this is not convenient for delicate payloads flying through a fluid, and the typical propulsion of a sounding rocket is a two-stage solid-propellant engine, with the following flight phases: ground-rail powered phase (of its own or external), free powered phase (aero-stabilised acceleration to maximum speed), ballistic flight to apogee (detected by an accelerometer or by a pressure-variation sensor), small-parachute ejection (a drogue, or just a streamer), and stabilised descent, with a main parachute deployment at some predefined lower altitude (around 100..300 m).
Rocket design basically aims at finding the mass of propellant needed to lift a given payload to a given apogee. Under the no-air assumption, the delta-v needed to vertically reach an altitude $H$ is $\Delta v = \sqrt{2gH}$, valid for $H << R$, and the trajectory is $z(t) = \Delta v t - \frac{1}{2} gt^2$, with a total flight-time of $\Delta t = 2\sqrt{2H/g}$. To this non-dissipative (or potential) delta-v, another delta-v is added empirically (constant, e.g. 1 km/s, or increasing with apogee), or computed with some aerodynamic air-drag model. A first estimate of propellant mass needed is then made with Tsiolkovsky rocket equation (3) with a final mass equal to the payload mass, $m_{pa}$, and an assumed rocket exit speed (e.g. $v_e=2500$ m/s for solid propellants); i.e. solving $\Delta v = v_e \ln((m_{pr}+m_{pa})/m_{pa})$ for the mass of propellant, $m_{pr}$, with the $\Delta v$ above-calculated; e.g. to lift a payload $m_{pa}=50$ kg to $H=100$ km altitude, $\Delta v = \sqrt{2gH} = 1400$ m/s (lets add $\Delta v=1$ km/s for air-drag), we solve $1400+1000=2500 \cdot \ln((m_{pr}+50)/50)$, obtaining $m_{pr}=130$ kg of propellant. A more detail design should account for the thrust-time law (e.g. neutral or dual thrust mode), nominal thrust (the time-integral of thrust, i.e. the total impulse, is defined by rocket mass and delta-v, so that only one can be fixed, the nominal thrust, or the burning time), additional mass due to the structure (chamber walls, nozzle, aerodynamic nose) and guidance, etc. Finally, the transient processes after ignition and after the whole solid propellant is burned, should be considered.

Exercise 4. Find the $\mu g$ period in a 100 m free-fall tower, and in a 100 km sounding-rocket flight, assuming there is no air drag.

Sol.: The time lapse in a 100 m fall is $\Delta t = \sqrt{2H/g} = \sqrt{2 \cdot 100/9.8} = 4.5$ s, whereas the up-and-down flight-time to 100 km altitude is $\Delta t = 2\sqrt{2H/g} = 286$ s (4.8 min).

Notice that $\mu g$ period can be doubled in a drop tower if some catapult device is used to throw the capsule from the bottom in an up-and-down flight under vacuum (but the shot must be very precise).

In practice, large drop towers are operated in drop mode only, and with ambient air, using two capsules, one inside the other, the inner holding the test cell for $\mu g$ experimentation, and the outer being streamlined for minimum air drag, with extra space to allow the relative axial displacement of the two capsules during free fall.

It should be noted that any spaceflight that returns to the surface, including sub-orbital ones, will undergo atmospheric reentry, but the aerodynamic heating is much less for a suborbital flight re-entering with a maximum speed of only 1 km/s, than for a deorbiting from LEO with a speed of 7 or 8 km/s.

**Air drag: ballistic coefficient**

Relative motion of a body within a fluid imposes a force on the moving object to compensate the loss of momentum and deflections in the fluid. This force is always considered split in two components, drag and lift, both in spacecraft dynamics within a fluid medium, and in aircraft dynamics (see Aerodynamic drag, aside).

Air drag is normally an undesirable effect, requiring energy to be dissipated by friction in the fluid, and introducing large uncertainties in trajectories; but air drag may be used for our advantage, as in parachutes, deorbiting spacecraft, and elimination of space debris. Both for reentry and launching on Earth, air drag may be said to be negligible above 90 km altitude.
Except at very low speeds, it happens that drag is roughly proportional to the square of speed, so that the non-dimensional drag coefficient $c_D$ is defined by $D = \frac{1}{2} \rho v^2 c_D A$, where $\rho$ is the fluid density far upstream, $v$ is the relative unperturbed speed, and $A$ is a reference area, which, for streamlined bodies, is chosen as the total wet area, or its projection in a plane parallel to the flow, whereas for blunt bodies $A$ is chose as the frontal projected area. Compressible flow increases the drag coefficient $c_D$, particularly in the transonic regime (drag divergence); air drag, and its fluctuation, is so large in that region, that transonic flight is always avoided, passing by quickly through if supersonic flight is wanted, or staying below (say $M<0.9$) for high-speed subsonic flight; for the same reason too, air guns are designed to shoot pellets (the traditional 4.5 mm calibre) at around 300 m/s to avoid loss of accuracy (no sonic bang is heard).

**Ballistic coefficient**

For steady motion, drag force must be compensated with thrust (or with weight, when falling with the terminal velocity); otherwise, air drag causes a deceleration on the body of mass $m$ according to $D = ma$, of magnitude $a = q/\beta$, where $q \equiv \frac{1}{2} \rho v^2$ is the dynamic pressure, and $\beta \equiv m/(c_D A)$ the ballistic coefficient; namely:

$$D = c_D \frac{1}{2} \rho v^2 A = ma \quad \rightarrow \quad a = \frac{q}{\beta}, \quad q = \frac{1}{2} \rho v^2, \quad \beta = \frac{m}{c_D A}$$

(13)

Notice that we here define $\beta$ in units of $[\text{kg/m}^2]$, but several other definitions of ballistic coefficient are in common use: $mg/(c_D A)$ in $[\text{Pa}]$, and their inverses $(c_D A/m, c_D A/(mg))$. The ballistic coefficient of an object is a measure of its terminal velocity, $v_t$, since, from the steady force balance in that case ($W = D$) one has $v_t = \sqrt{\beta g/\rho}$. In terms of the ballistic coefficient $\beta$ and launch speed $v$ (the speed at burnout), the altitude $H$ reached at apogee is:

$$H = \frac{\beta}{2k} \ln \left( \frac{k v^2}{\beta g} + 1 \right) = \frac{v^2}{2g} - \frac{k v^4}{4g^2 \beta} + O \left( \frac{1}{\beta^2} \right)$$

(14)

where $k = 0.613$ kg/m$^3$ is an aerodynamic constant; it can be seen that for large $\beta$ the vacuum result $H = v^2/(2g)$ is approached. Rockets with equal ballistic coefficients and equal burnout velocities will achieve equal altitude.

Drag computations are most often based on analytical or empirical formulations of the drag coefficient, and application of (13). Measurement of $c_D$ is based on experiments where the resistance force $D$ is directly measured in a wind tunnel, or the acceleration $a$ in free motion is measured; in any case, experimentation is really difficult at such huge speeds and high-altitude density, and computational fluid dynamics (CFD) with chemically reacting gases must be used. The reference area must be well defined (e.g. how it changes with flight attitude), and measured (it may be difficult to do it with uncontrolled-attitude objects like free-falling bodies, or dead satellites (and debris), which tumble along its trajectory).

Lateral forces on a body moving within a fluid, generally known as lift $L$ independently of the direction in the plane perpendicular to the advance speed, are modelled similarly to (13), i.e. $L = c_L \frac{1}{2} \rho v^2 A$. Even on perfectly axisymmetric bodies, lateral forces appear due to transient effects on the flow. To minimise such
disturbances, ballistic objects are usually spin stabilised. In subsonic flight $c_L$ and $c_D$ are roughly proportional to $\alpha$ and $\alpha^2$, respectively ($\alpha$ being the angle of attack, $\alpha<1$ implying $c_L>>c_D$, i.e. large aerodynamic efficiency, $L/D>>1$); however, at the high speeds in the supersonic and hypersonic regimes, both coefficients are proportional to $\alpha^2$ and thus $L/D~1$.

**Atmospheric density model**

A fluid density model is needed to relate drag with speed using (13). Density depends on temperature, pressure, and composition, which may vary in space and time. Several atmospheric models have been developed, the simplest being the exponential-density model, $\rho=\rho_0\exp(-z/z_e)$, defined by just two parameters: density at the planet surface (at $z=0$) $\rho_0$, and characteristic height $z_e$ (height at which $\rho/\rho_0=1/e=0.37$, or total height of an atmosphere with uniform density equal to the ground value).

Notice that an exponential-density model (for an ideal gas of uniform composition) implies that the atmosphere is isothermal, since the hydrostatic equation $d\rho/dz=-\rho g$ yields $p=\rho_0 g z e \exp(-z/z_e)$, and with the ideal-gas model, $p=\rho RT$, one has $T=g z_e/R=$ constant. The two parameters most often used to define $\rho_0$ and $z_e$ are the $p-T$ data at ground level $(p_0, T_0)$, from which we get $\rho_0=p_0/(RT_0)$ and $z_e=RT_0/g=p_0/(\rho_0 g)$. For Earth's atmosphere, if we chose $p_0=101.325$ kPa and $T_0=288.15$ K, we get $\rho_0=1.01\cdot10^5/(287\cdot288)=1.225$ kg/m$^3$ and $z_e=RT_0/g=p_0/(\rho_0 g)=287\cdot288/9.8=8.4$ km. However, if we are more interested on density at the stratosphere (where hypersonic flight takes place) than in the troposphere, a better fit is obtained using $\rho_0=1.4$ kg/m$^3$ and $z_e=7$ km, as can be seen in Fig. 6; i.e. we adopt:

$$\rho = \rho_0 \exp\left(-\frac{z}{z_e}\right) \quad \text{with} \quad \rho_0 = 1.4 \text{ kg/m}^3 \quad \text{and} \quad z_e = 7 \text{ km}$$

(15)

Notice that, with these values, we get $p_0=\rho_0 g z_e=1.4\cdot9.8\cdot7000=96$ kPa and $T=g z_e/R=9.8\cdot7000/287=239$ K.

![Fig. 6. Earth's atmosphere density versus altitude; comparison of ISA values (blue curve), with exponential model based on $p_0=101$ kPa and $T_0=288$ K (green line), and with our model (15) (red line).](image)

The assumption of uniform composition for Earth's atmosphere (homosphere, with approximate molar fractions $x_{N_2}=0.79$ and $x_{O_2}=0.21$) is very good from sea level (where main departure is the effect of water vapour, $<4\%$) to about 85 km altitude (that is why it was chosen as the limit of ISA model of 1976). Higher
up, molecular oxygen (O) starts to be significant; O is the major component from \( z = 180 \) km upwards (up to \( z = 500 \) km, where helium becomes preponderant).

**Mars atmosphere** is more difficult to model because spatial, diurnal, and seasonal changes are relatively more pronounced than on Earth.

**Air-drag coefficient**

Air-drag coefficient, \( c_D \), basically depends on body shape, and speed \( v \) (Reynolds number, \( Re = \frac{vD}{\nu} \), for viscous effects, and Mach number, \( M = \frac{v}{c} = \sqrt{\gamma RT} \) for compressibility effects). At low Mach number and high Reynolds number, the typical drag coefficient for a rocket-shape is \( c_D = 0.3 \) (\( c_D = 0.4 \) for a sphere). For a spherical body at low speeds, drag coefficient is given by Stokes law for laminar motion (creeping flow), \( c_D = \frac{24}{Re} \), i.e. inversely proportional to speed (very good up to \( Re = 10 \)). For \( Re > 10 \) it remains almost constant with a value of \( c_D = 0.3 \) (laminar boundary-layer flow with turbulent wake), applicable for \( 10^2 < Re < 10^5 \). At around \( Re = 2 \times 10^5 \) for smooth surfaces (\( Re = 0.8 \times 10^5 \) for rough surfaces), it rapidly falls to about \( c_D = 0.1 \) when the boundary layer becomes turbulent (what is known as drag crisis), and then increases to about \( c_D = 0.4 \) for \( Re > 10^6 \) as the turbulent wake widens. Higher speeds in compressible media make the drag coefficient to increase with speed (with Mach number) in a parabolic way to about \( c_D = 0.7 \) at \( M = 1 \), raising to a maximum of \( c_D = 0.9 \) for \( M = 2 \), remaining almost constant (a little below \( c_D = 0.9 \)) in the supersonic range \( 2 < M < 5 \) and then slowly growing towards \( c_D = 0.92 \) in the hypersonic regime (\( M > 5 \); Newton's model predicts \( c_D = 1 \)); in the **free-molecular regime** (i.e. when Knudsen number \( Kn \geq 1 \), a value around \( c_D = 2 \) is usually assumed. At present, practically all hypersonic flights (manned and unmanned) take place without propulsion, the exception being launching vehicles and research aircrafts; the first hypersonic flight was achieved by a second-stage rocket mounted on a V-2 in 1949.

Parachutes have a \( c_D = 0.4 \) to 0.8 depending on porosity (and to a lesser extent on canopy shape, suspension-line length, payload wake effects, Mach number, and Reynolds number). Porosity (area fraction of voids in the fabric and holes) rarely goes beyond 30%.

The problem of finding the optimal thrust-curve \( F(t) \) for a vertical 1D rocket trajectory \( z(t) \) with a flat Earth (constant gravity \( g \)), simple \( \rho v^2 \) air-drag model (i.e. with constant \( c_D \), in spite of the fact that it jumps from about \( c_D = 0.02 \) to \( c_D = 0.05 \) at around \( M = 0.9 \), and the exponential decaying atmosphere \( \rho = \rho_0 \exp(-z/\zeta) \), is known as **Goddard problem** (stated by him in 1919, and first solved, approximately, by Oberth in 1929). The equation of motion is \( m \frac{dv}{dt} = F - W - D = mv_y - \frac{1}{2} \rho v^2 c_D A - mg \), and the objective is to maximize \( z_{\text{max}} \), for given values of \( m_{\text{ini}} \) and \( m_{\text{fin}} \), a variational problem with no simple solution (e.g. by the discontinuity at burnout). The solution consists of an impulsive start (booster) followed by a longer-duration lower-thrust powered flight (sustainer), before the much longer ballistic coasting; in brief, the traditional 'dual thrust' strategy (boost/sustain), achieved by carefully chosen the solid propellant geometry.

**Descent and reentry**

We basically consider here the spacecraft propulsion needed to do a soft descent to the surface of a planet (or moon), either in the presence of an atmosphere or under vacuum (e.g. on the Moon or Mercury). Propulsion may be needed for deorbiting, and for landing safely (i.e. soft landing, in contrast with 'hard
landing' which often means destructive crash). Uncontrolled entries of spacecraft and their debris are mentioned, but impacts of natural celestial bodies (meteoroids), either through an atmosphere or under vacuum, are not included.

The acronym EDL (entry, descent, and landing) describes whole the sequence of events, and may be the best envelop to include different type of entries (from low orbits, or from high transfers), different types of descents (with an atmosphere, without, or even a flyby without landing), and different types of landing (hard landing, soft landing, or no landing but floating somewhere within the atmosphere, as in Vega mission to Venus). The word 'reentry' was coined for orbiting spacecraft coming back to Earth, and later extended to penetrating in any planetary atmosphere, but not used for descent to celestial bodies without atmosphere, like the Moon.

One may go in a direct high-elliptic orbit from the Earth to the Moon, as in the Surveyor program, arriving there at 2.6 km/s (in any case above Moon's escape speed, 2.4 km/s) and decelerating to a soft landing with retrorockets during the last minute, without marked entry and descent phases. But, for better control of the EDL path, it is preferable to spend some additional fuel to circularise the orbit to a LMO (low Moon orbit), by firing retrorockets at the approaching elliptic orbit when closest to the Moon (reducing the >2.4 km/s to the 1.7 km/s of a LMO), and then have time to perform a precise EDL sequence, initiating the entry by further reducing the speed (another propulsion delta-v), followed by a slightly controlled ballistic trajectory (with some lift on planets with atmosphere), until the landing phase where speed is reduced to about the 1 m/s that the shock absorbers may bear. But the entry-descent-landing sequence is poorly defined in planets without an atmosphere.

More clear cuts can be defined for entry into planetary atmospheres, where air-density and planet-gravity are the major constrains:

- The entry phase often lasts from a predefined altitude (e.g. 100 km, or from the retrorocket firing in low orbit), perhaps preceded by jettisoning of a cruise or orbiting service unit, to the deployment of aerodynamic decelerators (e.g. parachutes), if any; i.e. all except the final descent and landing. Most of the spacecraft kinetic energy \( E_k = \frac{1}{2} m v^2 \) is dissipated during the last part of this hypersonic entry phase; e.g. for the entry phase on Earth, the spacecraft decelerates from about 7.6 km/s at 300 km altitude (defined by gravity), to about 0.5 km/s at say 10 km altitude (defined by gravity and air-drag), but deceleration do not starts until about 85 km altitude (where air-drag becomes significant), and being maximum at about 45 km altitude. Fortunately, only a part of that energy (about 4% in the Shuttle reentry) heats up the craft and cause high skin temperatures and ablation (i.e. the part not convected and radiated outside), and that can be modulated by selecting a more or less lifting trajectory (and the shape of the craft). The heating is by the adiabatic compression in the shock wave (skin friction is only a minor contribution, <10 %). Pure ballistic reentry is the simplest solution for small capsules, but it requires massive thermal protection (almost 1500 kg out of the 6000 kg of Apollo reentry capsule), and is not good for large vehicles like the 100 t Shuttle. In any case, the high ballistic coefficient \( \beta = \frac{m}{c_D A} \) of blunt-body configurations yields a terminal velocity too-large for practical landing systems (except in high-lift gliders like the Shuttle), hence another descent phase is needed to further decelerate the vehicle.
The descent phase lasts from the deployment of the supersonic parachute (or any other active decelerator, like a ballute, or retrorockets), needed to slow down from about 0.5 km/s to a vertical speed of about 1 m/s at landing; this is usually achieved by increasing $c_{DA}$ in the ballistic coefficient $\beta = m/(c_{DA})$ of the falling body through the planet atmosphere. On the Space Shuttle, descent is in a long-haul gliding mode. Parachutes would break, and ribbons would be inefficient, if deployed at higher speeds.

The landing phase is just the touch down on the land (or the ocean), often including a final manoeuvre of retrorockets (or a final wired descent from another craft like in Mars Curiosity), deploying airbags, wheels and a landing runway, etc. The landing energy absorbers may be multi-use fluid-dissipation systems or one-shot solid crushable systems.

Reentry time plots usually have the time-origin at a predefined altitude (the entry interface, e.g. on Earth, the Kármán line, $z=100$ km; the value $z=122$ km=400 kft is often used), although one might chose the deorbit rocket-burn, since, after that, the spacecraft is committed to a particular landing location at a particular time, with minor range and endurance control capability by changing the lift-direction during descent (by banking to one side and the other side, i.e. doing S-shapes on the ground projection).

The descent from say 300 km altitude to 100 km altitude lasts some 1500 s, being almost the same for capsules and shuttles, and has little interest (almost ballistic), whereas below 100 km it is full of important events, which are different for high-lifting shuttles (which take another 1800 s to land), and low-lifting or no-lifting capsules (which only take about 500 s to land); hence, from the deorbit manoeuvre, it takes around 2000 s for a capsule to land, and around 3300 s for the Space Shuttle, as sketched in Fig. 7, where the evolution of several kinematic and dynamic variables are sketched for both, a typical launch (common to capsules and shuttles), and a typical reentry.
Fig. 7. Typical flight conditions during ascent (from lift-off), and during descent from LEO (with time-origin at $H=100$ km crossing): altitude $z$, speed $v$, down-range $x$, longitudinal g-load, dynamic pressure $q = \frac{1}{2} \rho v^2$, aerodynamic heating flux $\dot{q}$, and Mach number, $M$. Capsules land some 500 s after crossing the $H=100$ km line, with almost constant vertical speed (~250 m/s) until parachute deployment, whereas shuttles may take 1800 s to land.
Unmanned space probes have landed on the Earth (since Vostok-1K, also named Sputnik-5, in 1960, although suborbital reentry started with V-2 in 1944), on the Moon (since Luna-9 in 1966; Luna-2 in 1959 was intended to impact), on Venus (since Venera-8 impact in 1972), on Mars (since Mars-3 in 1976), on Saturn's moon Titan (Huygens probe landed in 2004), and Philae probe landing on comet 67P (Churyumov-Gerasimenko) in 2014. Most entry trajectories are planned to be direct from the approach orbit, i.e. without first circularising the orbit around the destination planet or moon, to save the extra delta-v required. ESA has developed an intermediate experimental vehicle (IXV) to learn controlled reentry from low Earth orbit (to be launched with Vega in 2015).

Manned reentry started in 1961 with Gagarin's trip on Vostok-1. The only non-Earth manned descent has been on the Moon: the six Apollo lunar landers put 12 astronauts on the surface (from 1969 to 1972). One of the unsolved problems of a manned trip to Mars is that the thin atmosphere there is not able to quickly and safely open a parachute or ballute to decelerate a large spacecraft (the heavier lander has been the 990 kg of Curiosity rover, but it is estimated that a return manned spacecraft will require to land some 40 000 kg); Low Density Supersonic Decelerators (LDSD) are being developed by NASA, to first decelerate from \( M=4 \) to \( M=2 \) with an inflatable drag device (6 m in diameter for large robotic spacecraft, inflated with hot pressurised gas; 8 m for crewed landers, inflated with ram gas), to be followed by a 34 m in diameter supersonic ring-sail parachute.

Reentry on Earth's atmosphere is dominated by the high speeds and associated high temperatures involved. As seen above, even a suborbital flight of just reaching apogee at \( H=100 \) km altitude has at 50 km height about \( v=\sqrt{2g\Delta z} = \sqrt{2 \cdot 9.8 \cdot 50 \cdot 10^3} = 990 \) m/s, i.e. \( M=3.0 \) (the environment temperature is \( T_{50}=271 \) K). But a craft coming from LEO has some \( v=7.7 \) km/s (i.e. \( M=24 \) in those conditions), a craft returning from the Moon some \( v=11 \) km/s (\( M=33 \)), the Stardust probe returned with stellar dust samples in 2006 at \( v=12.8 \) km/s (\( M=39 \), the fastest reentry to now; the stagnation temperature was around 25 000 K), and a return from Mars will be at \( v=15 \) km/s (\( M=45 \)). Atmospheric reentry is both a blessing (a means to get rid of the huge kinetic energy of orbiting spacecraft), and a curse (this huge energy dissipation, close to the spacecraft, causes very high temperatures on the body surface, usually above its melting point).

Reentry is the most dangerous phase of spaceflight, in spite of the formidable power developed during launch, as the statistics may prove (recall that the Apollo-1 accident in 1967 did not occur during launch but during a launch-rehearsal test, a month before launch, when a fire broke out in the pure-oxygen atmosphere at the command module, and the hatch could not be opened fast enough). The major problem on reentry is not the impact on the surface, which is easily softened with parachutes and/or retrorockets, but the very high temperatures associated to hypersonic flow through the atmosphere. To lower this aerodynamic heating, spacecraft air-drag should be maximised, by promoting pressure-drag against skin-friction-drag (as first discovered by NACA engineers Allen and Eggers in 1951); i.e. streamlined shapes get hotter than blunt shapes, because, in the latter, the bow shock-wave stands off the body surface.

**Deorbiting and reentry angle**

Deorbiting from LEO may take place without propulsion, since an orbiting object at say 300 km would naturally re-enter after a year if ballistic coefficient (mass-to-front-area-ratio) is about 1000 kg/m² (or after Spacecraft propulsion 42
a month if 100 kg/m²), because of the residual air drag; e.g. a 15 kg tool-bag lost in a spacewalk by STS-126 astronauts at 350 km altitude, orbited for more than eight months before burning during its reentry. But for satellites with altitude $H > 600$ km the natural orbit-decay period exceeds the 25 years limit presently impose to cope with the debris problem.

Quicker deorbiting may be achieved by increasing drag (reducing the ballistic coefficient), deploying appendages or inflatable devices (e.g. filling a balloon with helium), the most promising being electrodynamic tethers, which might become the common device to demise LEO satellites after their useful life. To guarantee a controlled reentry, however, deorbiting must be with some propulsion that presently is limited to the initial braking manoeuvre (firing rockets against the advancing direction, i.e. retrorockets), to achieve a certain reentry angle. When the intention is to demise the spacecraft, the landing target is usually chosen to be a region in the South Pacific Ocean (the 'spacecraft cemetery', 4000 km East of New Zealand); these reentry spacecraft start disintegrating at about 80 km altitude, and debris spread along and across the entry path.

The reentry flight path angle $\gamma$ is the angle between the local horizontal and the velocity vector of the spacecraft at a predefined altitude, usually the Kármán line on planets with atmosphere (e.g. 100 km altitude on Earth, 80 km on Mars). For a given entry speed, there is only a small range of entry angles (the entry corridor) that allow a suitable descent, because very low angles would cause bouncing (skipping out of the atmosphere for a while, perhaps several bounces without precise control), and too-steep entries (large $\gamma$) would cause excessive aerodynamic heating, excessive g-loads, and perhaps too-little time to decelerate and land in thin atmospheres. The required delta-v to deorbit from LEO is (from orbital mechanics):

$$
\Delta v = v_i \left[ 1 - \sqrt{\frac{2}{r_i/r_{ref} - 1}} \left( \frac{r_i}{r_{ref} \cos \gamma} \right)^2 - 1 \right]
$$

where $v_i$ is the speed at the initial circular orbit of radius $r_i$, and $\gamma$ the desired reentry angle at the reference radius $r_{ref}$, e.g. for $r_i=6370+300=6670$ km ($v_i=\sqrt{\mu/r_i} = 7.7$ km/s), and $\gamma=1.4^\circ$ at $r_{ref}=6370+100=6470$ km, the retrorockets must provide a delta-v of $\Delta v=115$ m/s (it is always in the range 100..150 m/s to achieve a reentry angle in the range 1..2°, low enough to prevent excessive deceleration and heating. If the Keplerian trajectory were extrapolated (i.e. no air-drag, not even the ground surface impact), the results shown in Fig. 8 would be obtained, which may be therefore applied to lifting and non-lifting bodies. Notice that the craft would impact at sea level after traveling around 14 000 km in 1800 s since the deorbit manoeuvre (only 2000 km in 500 s after crossing the Kármán line).
Fig. 8. Deorbit trajectory and flight-path angle obtained after a $\Delta v=115$ m/s retrorocket boost is performed in a circular orbit of $H=300$ km altitude, assuming no air drag. Time (left) and down-range (right) origins are fixed at retrorocket firing; sea level is at $z=0$; the reference Kármán altitude $H_{ref}=100$ km is marked).

This model may be applied to a first approximation for the case of reentry capsules from LEO (e.g. from $z=300$ km to $z=100$ km it takes 1500 s and covers $\Delta x=11\ 000$ km, and then from $z=100$ km to landing it takes 500 s and covers $\Delta x=2000$ km), but widely deviates in the $z<100$ km region for lifting bodies like the Space Shuttle, first because of its larger entry $L/D$ value (near $L/D=1$, whereas it is around $L/D=0.3$ in capsules), then because its much larger $L/D$ when gliding from $z=75$ km downwards (up to $L/D=4.5$). Consequently, from $z=100$ km to landing the Space Shuttle takes around 1800 s and covers $\Delta x=7000$ km, to land on a 90 m wide by 5 km long runway at 100 m/s.

When deorbiting from elliptical orbits, the braking manoeuvre should be performed at apogee, to save fuel. The entry angle must be greater for larger reentry speeds, to avoid bouncing; in Apollo, entering with 11 km/s, it was around $\gamma=6^\circ$ (up to $10^\circ$ in early Apollo flights), i.e. a steeper descent. Maximum heat flux increases almost linearly with $\gamma$, with triple value at $\gamma=6^\circ$ than at $\gamma=1^\circ$, but the total heat absorbed reduces to a half because of the shorter path at large entry angles. For entries with very large $\gamma$ as for meteoroids (almost perpendicular to Earth's surface), the heating and deceleration is abrupt and at low altitude (down to 5 km), almost like hitting a water surface; e.g. on 2013-02-15 a 15 m meteoroid entered at 17 km/s fell over Chelyabinsk (Russia), bursting in air at about 20 km altitude (15..25 km), with its shock wave causing windows and wall damage, and 1000 people minor-injured; it was the largest meteoroid impact after the Tunguska event of 1908 (a 100 m asteroid that knocked down some 80 million trees in Siberia), and happened to occur the same day that a 45 m asteroid was expected to skim across the Earth (later named Duende, it did grazed at 28 000 km, below GEO altitude, and was smaller and elongated, of $20\times20\times40$ m$^3$).

Do not confuse the reentry angle ($\gamma$) with the angle of attack ($\alpha$, roughly body axis over apparent wind), which is much larger during reentry, to increase drag and slow down the craft. On gliding bodies like the Shuttle, the reentry flight path angle slowly decreases from $\gamma=1.5^\circ$ at 120 km altitude to $0^\circ$ at landing as the density increases (the angle of attack is kept at $\alpha=40^\circ$ during the hypersonic phase ($c_D\approx c_L$), decreasing to $\arctan(c_D/c_L)\approx\arctan(1/5)=11^\circ$ during the subsonic approach), whereas for non-lifting bodies it increases from some $20^\circ$ to $90^\circ$.  

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Reentry phases

The sequence in a reentry from LEO is:

- From LEO (say 300..800 km altitude range) to about 110 km altitude, there is free-molecular flow (particle mean-free-path $\lambda$ greater than object size $L$, i.e. Knudsen number $Kn=\lambda/L>>1$, best described by statistical mechanics), with negligible air-drag (no shock wave observed). The spacecraft makes use of attitude control thrusters to change from the deorbit boost configuration (rockets firing forward), to the entry configuration: bottom-first with an angle of attack $\alpha=20^\circ$ for capsules, or nose-first $\alpha=35^\circ..40^\circ$ for the Shuttle).

- From 110 km down to 90 km, there is a transition flow regime (best described with Boltzmann equation). Still negligible air-drag and negligible air-heating. The trajectory from LEO to 90 km altitude is purely ballistic, with no lift or drag, advancing speed of about $v_0=7.6$ km/s (the LEO speed minus the delta-v for reentry). The attitude (angle of attack) is maintained by thrusters or by spin stabilisation. In this region, the descent rate $v_0\sin \gamma$ is almost constant (dz/dt from Fig. 8), since for an entry angle $\gamma=1.5^\circ$, the falling speed is $v_z=7.6\cdot\sin1.5^\circ=0.2$ km/s (only for deorbiting from LEO; higher for deorbiting from higher orbits).

- From 90 km down to 50 km, there is a continuous fluid media, with hypersonic flow of a blunt body (high angle of attack if lifting body), developing a strong shock wave, highly compressing the gas, which can no longer be assumed ideal-gas. Aerodynamic heating is most important, with a peak around 55 km altitude for reentry capsules from LEO (at 45 km for capsules coming from extra-terrestrial space, and 60 km for the Space Shuttle), producing air ionisation that causes a radio-communication blackout. In most cases, a small lift on the craft is wanted for stability, being achieved by offsetting the centre of mass, what imposes a certain angle of attack and a lift force that can be oriented at will by rotating the spacecraft around its axis, giving some control on flight-path angle and cross-range velocity (non-lift uncontrolled entry yields higher loads and range uncertainties). Except for the Space Shuttle, with a high $L/D$ ($L/D=1$ in the hypersonic regime, $L/D=2$ in the supersonic regime, and $L/D=5$ in the subsonic regime), all landing capsules have a rather low lift-to-drag ratio ($L/D=0.3..0.4$). The momentum balance along this unpowered path is roughly $mdv/dt=mg\sin\gamma-D$, and the lateral force balance $L=m(g-v^2/R)\cos\gamma$, where $R$ is Earth radius ($H<<R$). Using the exponential-density model, one can get the advancing speed in terms of altitude, i.e. $v(z)$, and the corresponding gliding angle $\gamma(z)$, (path angle to the horizontal); namely:

$$v = \frac{v_i}{\sqrt{1 + \frac{c_l}{c_D} \frac{\rho_0 g R}{2 \beta} \exp \left( -\frac{z}{z_e} \right) }}, \quad \gamma = \frac{2}{c_l} \frac{R}{c_D z_e \left( \frac{v}{v_i} \right)^2}$$

(17)

where $v_i$ is the initial speed at entry ($v_i=7.6$ km/s), $\beta=m/(cDA)$ the ballistic coefficient, and $z_e=7$ km was defined in (15). The maximum aerodynamic heating flux and corresponding altitude can be estimated by the empirical fitting:

$$\dot{q} = C v^3 \sqrt{\rho/r_{nose}} \quad \text{with} \quad z_{\dot{q}_{\text{max}}} = z_e \ln \frac{\rho_0 z_e}{\beta \sin \gamma}$$

(18)
where \( C=10^{-4} \text{ kg}^{1/2}/\text{m} \) is an empirical constant for Earth reentry, \( \rho=\rho_0 \exp(-z/z_e) \) is the exponential atmosphere model (15), \( r_{\text{nose}} \) is the radius of the craft nose, \( \beta=m/(cDA) \) the ballistic coefficient of the craft, and \( \gamma \) the path angle. For instance, for an empty Space Shuttle \((m=70 \text{ 000 kg})\) with a 2 m nose radius, a path angle \( \gamma=1.5^\circ \), a ballistic coefficient \( \beta=1400 \) estimated with \( cD=0.5 \) and \( A=100 \text{ m}^2 \) (projection at an angle of attack of \( 30^\circ \)), we get from (18) that the heating peaks at \( z_{\text{max}}=40 \text{ km} \) (actually it peaks at 60 km), and we get from (18), with \( v=3 \text{ km}/\text{s}, \dot{q}_{\text{max}}=0.4 \text{ MW}/\text{m}^2 \), i.e. a typical heat flux that lasted up to 300 s on the Space Shuttle (on reentry capsules it is a shorter but stronger peak; up to 3 MW/m\(^2\) in Apollo). Notice that the Space Shuttle has a low vertical speed, about 20 m/s in the 60..80 altitude range, in the equilibrium gliding mode, attaining a maximum nose temperature of 2000 K (on Apollo, 2400 K were reached on its ablative shield).

At around 50 km altitude, maximum deceleration takes place (maximum g-force, and maximum dynamic pressure), with a narrower and stronger deceleration peak for gliding bodies than for capsules. The Space Shuttle reaches 50 km altitude, some 1000 s after crossing the 100 km Kármán line, travelling at about 2.5 km/s (about \( M=8 \)), still some 500 km off range, whereas Apollo reached the 50 km altitude just 100 s after crossing the 100 km line, still moving at about 9 km/s (about \( M=30 \)). The maximum aerodynamic deceleration and corresponding altitude can be estimated by:

\[
a_{\text{max}} = \frac{v^2 \sin \gamma}{2e z_e} \quad \text{with} \quad z_{\text{a,max}} = z_e \ln \frac{\rho_0 z_e}{3 \beta \sin \gamma}
\]

where \( e=\exp(1)=2.7 \); e.g. for the Space Shuttle, with the values used above, the acceleration peak occurs at \( z_{a,\text{max}}=30 \text{ km} \), and we get from (19), with \( v=1.5 \text{ km}/\text{s}, a_{\text{max}}=30 \text{ m}/\text{s}^2 \), i.e. a typical deceleration corresponding to a g-force of 4g (10g were reached during Apollo reentry from the Moon). The exponential-density model also predicts the maximum g-force speed, \( v_{g,\text{max}} \), being only dependent on entry speed, \( v_0 \), with \( v_{g,\text{max}}=v_0/e^{1/2}=0.61 \cdot v_0 \), and similarly for the maximum heat-flux: \( v_{q,\text{max}}=v_0/e^{1/6}=0.85 \cdot v_0 \) (larger than the former because \( \dot{q} \propto v^3 \) whereas \( a \propto v^2 \)).

At around 10 km altitude or below, when the speed is below 500 m/s (\( M<2 \)) more efficient decelerators must be activated on reentry capsules (shuttles dissipate during a much longer period, in gliding mode), usually by passive aerodynamic means (deploying supersonic parachutes in sequence or in clusters), perhaps aided by active propulsion means (retrorockets). For the falling capsule to know when to deploy the chutes (and other time events), the on-board computer compares a pre-loaded trajectory plan (based on the atmosphere model and body model) with accelerometer measurements (and its integration to estimate actual speed). The strong pull on the craft when parachute opens can be used to get rid of the protective heat-shell from the lander. The need for this deployed decelerators is double:

- Entry spacecraft have large ballistic coefficients, \( \beta=m/(cDA) \), because they must be compact and strong to bear air-drag forces (extended parts would break apart during the descent, soon after crossing the 90 km altitude where air-drag starts), and this implies too large vertical speeds, incompatible with any practical landing system. Notice that with a descent speed of 500 m/s, it would take just 20 s to crash from this 10 km altitude.
Entry spacecraft have compact shapes that become unstable in the transonic regime, hence, parachutes must be deployed in the low supersonic regime, say $1.4<M<2.5$, to stabilise the transonic crossing.

For the first manned reentry (1961, Yuri Gagarin on Vostok-1), the spacecraft's automatic systems brought it into the required attitude, and liquid-fuelled retrorockets fired for about 42 seconds. In case of reentry engine malfunction, the spacecraft was designed to descend within 10 days due to orbital decay, but the actual orbit differed from the planned one, and would not have allowed descent until 20 days post-launch while the life support system was designed to function for 10 days as a maximum. Pure spherical entry vehicles were used in the early Soviet Vostok and Voskhod capsules (and in Soviet Mars and Venera descent vehicles).

The Apollo Command/Service Module used a spherical section fore-body heat-shield with a converging conical after-body. It flew a lifting entry with a hypersonic trim angle of attack of $-27^\circ$ ($0^\circ$ is blunt-end first) to yield an average $L/D=0.37$. This angle of attack was achieved by precisely offsetting the vehicle's centre of mass from its axis of symmetry. Even these small amounts of lift allow trajectories that reduce peak g-force from the purely ballistic flight of $8.9g$, to a much bearable $4g$, as well as greatly reducing the peak reentry heat. The command module used eight parachutes during the descent phase on reentry: two drogues for initial stabilization, and three pilot chutes deploying the three main chutes (the system was sized to survive single parachute failures).

The first space station, Mir ($m=143\cdot10^3$ kg), was deorbited in 2001 (after 15 years of use) by several propulsion manoeuvres with a Progress rocket sent on purpose, after waiting many months for natural orbit decay to about 220 km altitude. The initial breakup began at about 95 km, when aerodynamic forces tore off the solar panels. At 85 km, all peripheral pieces were torn away, and the main modules began to buckle. A first delta-v had lowered the perigee a little, a second burn at the following apogee further lowered the perigee to 165 km altitude, and the third and last delta-v, performed also at apogee two orbits later, forced a quick reentry, crossing the 100 km Kármán line some 35 minutes later, and burning down some 15 min after, with debris spread $\pm1500$ kilometres along track and $\pm100$ kilometres laterally.

The Space Shuttle ($m=70\cdot10^3$ kg empty, 100 t fully loaded) has been the only reusable reentry spacecraft, with 135 entries in its 30 years of use, one of them catastrophic (STS-107 Columbia in 2003). As the vehicle encountered progressively denser air, it began a gradual transition from spacecraft to aircraft. The Shuttle had three landing places to choose for nominal landing, but, after choosing one, there was just one chance to land (no propulsion power for an aborted landing)

It might appear that reentry, being a non-propulsive trip 'coming home', should be safer than being launched on top of a huge pyre of fuel, but the contrary is true (more fatal accidents have occurred on reentry than on launching), mainly because of the additional controllability offered by powered flight (there are several rescue procedures during launch, but only one during entry: the switch to pure ballistic mode). Following Columbia accident during reentry, concerns about possible damage to flying aircraft due to widespread
debris from a reentry accident arose (normal launch and descent operations of spacecraft and sounding rockets are always protected by air-traffic restrictions on spot).

**Landing**

Landing, in general, refers to the final phase of flight in which the craft touches down the surface of the planet or moon, either solid (land) or liquid (ocean); another possibility might be a descent to a mid-height in a gaseous atmosphere, stopped by buoyancy forces. We only deal here with landing capsules (Shuttle landing has been described above). There are several requirements on landing: first of all is a limited vertical speed for soft landing, and second is limited displacement from the expected touch-down site, which are different in the advancing direction than laterally; i.e.:

- **Cross-range** is the distance either side of a nominal reentry track that may be achieved by using the lifting properties of an entry spacecraft.
- **Down-range** is the horizontal distance travelled by a spacecraft from entry to landing, or the horizontal distance travelled from the launch site. The origin may be at the launch site or nominal landing site, or at the projection of the spacecraft position when the deorbit rocket fires, or when crossing a reference altitude (e.g. the Kármán line).

To avoid destructive crashing, relative speed must be previously lowered during the descent phase, either by a large air-drag on planets with atmosphere, or/and by retrorockets; e.g. on Earth reentry, Apollo capsules landed on sea solely with parachutes, Soyuz capsules land on ground with parachutes and retrorockets, Space Shuttle landed on a runway. **Curiosity** landed on Mars hanging with cables from a mother descent vehicle that used retrorockets. **Huygens** was an atmospheric entry probe developed by ESA and carried on board the USA **Cassini** Saturn's orbiter, which landed successfully on **Titan** moon in 2005. **Philae** landed successfully on comet **67P**, after a couple of rebounds on the surface, since it approached at about 1 m/s but the grasping devices failed to anchor it to the surface (local gravity on this small mass was just $10^{-3}$ m/s²).

Safe landing speed on a planet is 1.2 m/s, but up to 3 m/s are used when landing with retrorockets to minimise the operation time close to the surface regolith (which cause a lot of dust that may damage equipment). Higher landing vertical-speeds are tolerated by airbag devices (first used in URSS **Luna** landers in 1960s), working in a vented fashion like in our cars, or being closed bags but allowing several bounces on the surface; of course, the larger g-forces might cause some damage (like in our car's airbags), but preserving most important parts of the spacecraft.

The **Surveyor** program (1966-1968) sent 7 robotic spacecraft to the Moon surface, to demonstrate soft landing in preparation of the crewed **Apollo missions**. The launcher, **Atlas-Centaur**, injected the craft directly into a trans-lunar flight-path, i.e. an impact trajectory approaching Moon surface at 2.6 km/s, decelerated first to about 100 m/s by a main solid fuel retrorocket, which fired for 40 seconds starting at an altitude of 75 km above the Moon, and then was jettisoned 11 km from the surface. The remainder of the trip to the surface, lasting about 2.5 minutes, was handled by smaller doppler-radar units and three vernier engines running on liquid fuels fed to them using pressurized helium, that stopped 3.4 meters to the surface where the falling speed was almost zero; in the subsequent free fall the craft arrived at 3 m/s at the surface.
Surveyor 6 was the first spacecraft planned to lift off from the Moon's surface. Surveyor 3 was the first spacecraft to unintentionally lift-off from the Moon's surface, which it did twice, due to an anomaly with Surveyor's Landing Radar which did not shut off the vernier engines but kept them firing throughout the first touchdown, and after it. Apollo landed on the Moon from a 110 parking LMO in two phases: first a delta-v made the orbit elliptic, with 15 km at perilune, and then a powered descent (including sufficient hover time at low altitude to select the exact landing site), using a descent propulsion system (DPS) of 45 kN thrust (throttleable between 10 % and 60 %, and gimbalable) with four tanks of hypergolic propellant, a 50/50 mix of N₂H₄/UDMH and N₂O₄ (8200 kg in total), pressurised by a 10 MPa helium tank (22 kg in total). Smaller rockets (16 kN) using the same propellants were used in the ascent propulsion (4500 kg of ascent mass).

Notice that the g-force on astronauts has the same direction during ascent and descent for capsules (pushing them against their seats, but no so much on the Space Shuttle. The most successful reentry capsule to date is Soyuz, being designed to minimise reentry mass (reentry lasts only minutes), with all auxiliary living systems into a separate jettisonable section joined by a hatch), and to minimise reentry volume (almost spherical shape, with a small banking to have a slight-lift to limit maximum g-load.

NOTE. The specific force or g-force is the non-gravitational force per unit mass, causing stress/strains on the object, and it is what measures an accelerometer. It can also be named equivalent-weight acceleration, or non-free-fall acceleration (we use here the symbol aₐₙ). Real total acceleration for vertical motion is $a=\left(F-W-D\right)/m=aₐₙ-g$, but horizontally would be $a=\left(F-D\right)/m=aₐₙ$, e.g. before lift-off (when $v=a=0$), is $aₐₙ=1g$ (one-times-Earth-gravitational-acceleration, a multiplication of a number by a variable); hovering with $v=0$ after lift-off ($F=W$) is with $a=0$ and $aₐₙ=1g$ too, but, when accelerating upwards with say $a=10$ m/s², it is $aₐₙ=2g$ (i.e. as if body-weight was doubled). The upper safe limit for manned return to Earth from LEO or lunar return is 10g (up to 20g for an impact), but design is for a maximum g-load of $aₐₙ=4g$. The g-limit is to avoid lung crushing. Weightlessness is $aₐₙ=0g$, named 'zero-g' (where now 'g' is a text label, not symbol), although on LEO there are residual forces (air drag, gravity gradient, vibrations from other equipment...) of magnitude slightly lower than $aₐₙ=10^{-6}g$ (without fixed direction and modulus, often said 'g-gitter'), and hence the g-force environment in LEO is better known as microgravity (µg) instead of weightlessness. On ground also, g-load is not a constant ($aₐₙ=1g=9.80665$ m/s² used as a standard), since actual g-value depends on location, but the relative change can be neglected under most circumstances. G-force must not be confused with the load factor used in aeronautics, which is the ratio of aircraft-lift to aircraft-weight, $n=L/W$, an appropriately signed non-dimensional scalar (often incorrectly written 'g-units', e.g. $n=3g$), although both terms (g-force and load factor) are often used undistinguishably in astronautics. Notice that very high g-loads only appear during aerodynamic deceleration (reentry, abrupt manoeuvres), since the acceleration due to the propulsion system is limited to a few gees (the engine specific thrust, $a=F/m$, in the limit).

**LAUNCHERS**

In this context, a launcher is a vehicle used to carry spacecraft from ground to orbit (i.e. to a trajectory not coming back to ground at once). All present launchers use expendable rocket engines (expendable launchers)
vehicles, ELV), but some air-breathing engines have already been tried as their first stage, i.e. launching a rocket from an airplane (e.g. Pegasus, SpaceShipOne, SpaceShipTwo, and modified missiles carrying nanosatellites, launched from high-altitude fighter-aircraft). Reusable launch vehicles (RLV) may refer to reusable boosters (like the first stage of Falcon 9), or future spaceplanes. Launchers to operate from other planet or moon might be considered included, but at present, we have only lifted off from the Moon, where the reduced gravity and lack of atmosphere makes the launcher much smaller and distinct to Earth launchers (the 4700 kg ascent lunar module LM was ‘launched’ from Moon’s surface with a $F=16$ kN rocket, i.e. with a thrust-to-weight ratio of $16/(4.7\times9.8)=0.35$ on Earth, but $16/(4.7\times1.6)=2.1$ on the Moon). Sounding rockets and other suborbital rockets are not usually considered launchers.

Huge amounts of energy are required to take a body from ground to orbit (a minimum of 34 MJ/kg to LEO, 58 MJ/kg to GEO) and without the possibility (at present) to recover any amount of such an investment. Contrary to common believe, this huge energy is not to lift the body but to make it go around, since, according to (5), it only costs $\Delta(-\mu/r)=-\mu/(R+H)-(-\mu/R)=-59-(-63)\approx 4$ MJ/kg to lift a body to say $H=400$ km LEO (a simpler estimate is $\Delta_{\text{pot}}=gH=9.8\cdot400\cdot10^3=4$ MJ/kg), whereas accelerating from rest to 7.7 km/s orbit speed needs $v^2/2=30$ MJ/kg. Overcoming air drag may add almost 10 MJ/kg to the expenses.

It might be thought that this energy-expenditure is not a problem, since we pay less than 2 €/kg for our car fuel with a heating value around 50 MJ/kg (i.e. the energy to take a 70 kg person to the ISS would cost $70\cdot34\cdot2/50=95$ €), but actual costs are many orders of magnitude larger: some 10 000 €/kg for any payload to LEO, instead of $34\cdot2/50\approx 1$ €/kg. And this is minimum bulk price; the cost to launch a 1 kg CubeSat is about 50 000 €. The basic reason for this meagre launcher efficiency is that we cannot launch just the payload but must lift off all the propellants and engine structures; e.g. the Apollo capsule was about 4 m in size and of 6000 kg, but its launcher, Saturn V, was 110 m long, weighting 2800 t from which only 30 t reached lunar orbit (the CSM), and only 15 t landed on its surface (the LM). Furthermore; both fuel and oxidiser must be carried on in rockets, since we do not master (yet) hybrid engines that could take oxygen from the air while there is plenty (perhaps up to 40 km altitude), and save oxidiser only for the near-vacuum propulsion phase (e.g. carrying LOX as oxidiser and using it progressively to compensate air thinning).

Launchers must have thrust-to-weight ratio greater than unity ($F/W>1$), must provide huge delta-\nu (more than $\Delta\nu=8$ km/s, whereas typical space rockets yield $\Delta\nu<<1$ km/s), and must do it in a relatively short time (about 10 min, instead of the >10 years of typical spacecraft operation). Only a minor part (or nothing) of the launcher is recovered (e.g. the empty solid boosters), the rest being left to burn on reentry. Most launchers take off from land, but a few did it from sea or from a flying aircraft.

According to Tsiolkovsky rocket equation (3), the minimum initial-to-final mass ratio for a launcher, $m_{\text{ini}}/m_{\text{fin}}=\exp(\Delta\nu/v_e)$, to reach LEO ($\Delta\nu\approx8000$ m/s without air drag, 9400 m/s all included), with a typical exhaust speed for solid rockets of $v_e\approx2500$ m/s, is $m_{\text{ini}}/m_{\text{fin}}=\exp(8000/2500)=25$ ($m_{\text{ini}}/m_{\text{fin}}\approx6$ for the best cryogenic rockets with $v_e\approx4500$ m/s), whereas in practice, it is $m_{\text{ini}}/m_{\text{fin}}=777/21=37$ for Ariane V, and it was $m_{\text{ini}}/m_{\text{fin}}=2030/25=81$ for STS (but, if the orbiter is included, $m_{\text{ini}}/m_{\text{fin}}=2030/100=20$), and $m_{\text{ini}}/m_{\text{fin}}=2800/120=23$ for Saturn V. Typical payload mass-fractions at launch are (as for year 2012) about $m_{\text{fin}}/m_{\text{ini}}=1/37=3\%$ to LEO, 1% to GEO, 0.5 % to Mars orbit, and 0.05 % to Mars Surface.
All present launchers are multistage, i.e. consist of two or three propellant reservoirs that are discarded once emptied (often with their own engines), because a single stage to orbit (SSTO) launcher is presently impracticable because of the minimum structural ratio for rocket engines (dry mass over propellant mass) achievable nowadays, around $\lambda = m_{\text{dry}}/m_{\text{pro}} = 0.2$. A SSTO rocket would only yield a delta-v of $\Delta v = v_e \ln((m_{\text{dry}} + m_{\text{pro}} + m_{\text{pay}})/(m_{\text{dry}} + m_{\text{pay}}))$, which, even the best chemical rocket (with $v_e \approx 4500$ m/s) and without any payload, would yield $\Delta v = v_e \ln((1+\lambda)/\lambda) = 4500 \cdot \ln((1+0.2)/0.2) = 8100$ m/s, less than the 9400 m/s needed, requiring at least a second stage (shedding some dry mass as propellant is used).

Launcher sites impose some constraints on minimum orbit inclination and safe ascent path:

- **Baikonur** is at 45.9ºN, but has to launch northbound to avoid flying over China, so that nominal orbit inclination (the minimum without added delta-v) is $i=52^\circ$, chosen by the ISS.
- **Kennedy Space Center (KSC, formerly Cape Canaveral)** is at 28.6ºN, but has to launch northbound to avoid the Bahamas (and Cuba to the South).
- **Kourou** is at 5.2ºN, and has to launch from North to East to avoid Brazil, but its safe flight space is 95º wide.
- **Jiuquan** is at 41ºN; it was used to launch the Shenzhou manned missions, but the ascent path is over some populated areas.
- **Xichang** is at 28ºN, but it is near populated areas (some downrange homes were damaged during lift-off of Chang'e 3 probe to the Moon in 2013).

**Launcher trajectory**

From the theoretical point of view, without air drag, the most efficient launch would be a horizontal take-off (HTO) single-stage-to-orbit (SSTO) trajectory, with most of the propulsion phase supported on ground (i.e. using electromagnetic propulsion on very long horizontal rails, or using detachable rockets on sleds), providing the $\Delta v = 8050$ m/s needed for the spacecraft to follow a ballistic trajectory (an elliptic orbit) starting horizontally at sea-level and reaching apogee (e.g. at $H=400$ km) also horizontally, where a small boost of 118 m/s would finally make the LEO circular. Unfortunately we cannot use such space guns to put spacecraft to orbit, not because of too-high g-loads, which can be reduced by lengthening the propulsion phase on ground, but because of the unsurmountable drag when facing air at 8000 m/s (with a sonic speed $c=340$ m/s at sea level, that means a Mach of $v/c=24$; a fighter plane needs full power to just go over $M=1$ at sea level).

The first part of the trajectory, where the air is thicker, must be crossed at relative low speed, and preferably upwards to reach thinner air layers as soon as possible. But once the launcher has to support its own weight after lift-off, and during a prolonged period (to go through the air at not too high speed), the need of propellant escalates: to provide a thrust greater than its weight, one needs to exhaust at a rate $m_{\text{pro}} = F/v_e > W/v_e = (m_{\text{dry}} + m_{\text{pro}} + m_{\text{pay}})g/v_e$, and the longer the boost the larger the $m_{\text{pro}}$ to be lifted. As a trade-off, practically all launchers lift off vertically at a quick peace (all using chemical propellants of high density to minimise volume, i.e. no LH2/LOX for boosters), turning sideways at around 40 km altitude (slowly while ascending; e.g. at 50 km altitude air density is 1/1000 of that on ground, and >99.9% of the atmosphere-mass is left below), and accelerating almost horizontally from that on, as already presented in...
Fig. 7 above. Recall that, leaving aside air drag, from the 34 MJ/kg that the launcher must add to the payload to reach LEO, only 4 MJ/kg are devoted to raising it against gravity, and 30 MJ/kg are needed to accelerate it to an orbiting speed of 7.7 km/s.

Notice that, during ascent, outside air pressure falls rapidly, and appropriate venting must be provided at the payload fairings to avoid the build up of overpressure inside, which may contribute to structural loads, and have detrimental effects during payload delivery later on. Similarly, all payloads must be provided with venting holes to get rid of the air trapped on ground, which might distort or even spoil delicate items like MLI. As a general rule, payload vents are designed with characteristic times in the order of seconds (1..10 s), whereas fairing vents are designed with characteristic times around 1 minute; in that way, venting holes in the payload always work in the subsonic range, whereas venting holes in fairings become sonic (choked) at high altitude (z>10 km).

The launcher is further optimised by recovering at each stage the rocket engines and structure (all except for the propellants used, of course), and by taking advantage of Earth rotation entrainment speed when possible (i.e. launching due East from a launch site close to the Equator). Launching from a high-altitude (e.g. at 2.5 km, already a 25 % of the atmosphere is left below) is impractical because the heavy loads to be transferred demand sea-shipping and a near sea-shore launch-site location.

Table 2 gives a short summary of events during launch to LEO, which basically consist of around 2 minutes of first-stage burning, to clear off the atmosphere almost vertically, immediately followed by around 8 minutes of second-stage burning to accelerate it almost horizontally, then some 20 minutes of ballistic flight to reach LEO altitude, plus a small final boost to circularise the orbit (or larger for higher transfer orbits); the multi-stage rocket system can be easily appreciated in Fig. 9. During the turning from vertical to nearly horizontal pitch at around 50 km altitude, the angle of attack (angle of incidence of air relative to launcher axis) is almost zero; the turning naturally progressing by contribution of the weight (gravity turn). During the ascent phase, the payload is protected from aerodynamic forces and associated heating by a fairing, usually detached soon after crossing the 100 km altitude Kármán's line. As soon as a part of the vehicle is no longer needed, it is discarded, to maximise subsequent thrust effect. Additional launch and ascent details for Space Shuttle are presented in Fig. 15 and Table 4, below.
Table 2. Typical events in a space launch to LEO after take-off time (TO).

<table>
<thead>
<tr>
<th>Time</th>
<th>Height</th>
<th>Speed</th>
<th>Mach</th>
<th>Horiz. dist.</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>TO+45 s</td>
<td>7 km</td>
<td>0.3 km/s</td>
<td>1</td>
<td>3 km</td>
<td>Possible condensation clouds</td>
</tr>
<tr>
<td>TO+1 min</td>
<td>10 km</td>
<td>0.6 km/s</td>
<td>2</td>
<td>8 km</td>
<td>Maximum aerodynamic forces</td>
</tr>
<tr>
<td>TO+2 min</td>
<td>50 km</td>
<td>1.5 km/s</td>
<td>4</td>
<td>50 km</td>
<td>First stage rockets discarded</td>
</tr>
<tr>
<td>TO+5 min</td>
<td>110 km</td>
<td>3.5 km/s</td>
<td>11</td>
<td>150 km</td>
<td>Fairing detachment</td>
</tr>
<tr>
<td>TO+9 min</td>
<td>150 km</td>
<td>6.5 km/s</td>
<td>20</td>
<td>500 km</td>
<td>Second stage (main rockets) cut-off</td>
</tr>
<tr>
<td>TO+10 min</td>
<td>140 km</td>
<td>7.0 km/s</td>
<td>NA</td>
<td>1 000 km</td>
<td>Third stage (or 2nd-stage re-ignition)</td>
</tr>
<tr>
<td>TO+30 min</td>
<td>300 km</td>
<td>7.7 km/s</td>
<td>NA</td>
<td>10 000 km</td>
<td>Final orbit circularisation.</td>
</tr>
</tbody>
</table>

Launch vehicles need some thrust throttling and deflection capabilities to compensate for large variations in aerodynamic pressure ($\Delta p = \frac{1}{2} \rho v^2 < 50$ kPa for structural reasons), to avoid large accelerations in human flight (g-load limited to 3g for some-minutes exposure, because larger values can only be withstood during seconds), to compensate wind entrainment, and to fine-tune the trajectory. Usually, some vector-control of thrust (engine gimbal) is used, to avoid the need of extra rockets for attitude control. Besides, launcher nozzles must face varying expansion conditions from the ground to outer space (over-expanded at take-off and under-expanded at high altitude). Notice that during ascent, mechanical loads grow with $D \propto \rho v^2$ whereas thermal load grow with $Dv \propto \rho v^3$, and hence, the heat peak occurs at higher speeds than the g-load peak (both, during launch and during reentry); see Fig. 7, above.

The electrical power needs of a launcher are modest (only to run the communication and data-handling subsystems, as well as sensors and actuators), and it is usually provided by batteries, although fuel cells were used in the Space Shuttle due to its long mission-duration and recoverability. Current launch vehicles rely on the Global Positioning System (GPS) for information on its current position, velocity, and attitude. Launcher trajectories should also be optimised for easy communications with ground support, what may impose a higher-altitude trajectory than optimal for propulsion.

Crewed launchers usually have a launch escape system (LES), i.e. a small solid-propellant rocket capable of pulling the crewed capsule away from the main vehicle in an aborted launch, first used on a test of a Mercury capsule in 1959. In Saturn V (Fig. 10), the LES was jettisoned at an altitude of 100 km, 30 s after ignition of the second stage. An emergency use of an LES occurred during the launch of Soyuz T-10-1 on 1983; the main rocket caught fire just before launch, and the LES carried the crew capsule clear, seconds before the rocket exploded (the crew were subjected to 14..17g for five seconds, the capsule reached an altitude of 2 km, and landed 4 km from the launch pad.)
Safety around the launch pad and ascent path is another issue, and an area several kilometres wide and hundreds kilometres along the ground track (while large amounts of propellants and debris would fall without disintegration) must be considered in danger.

Launch windows and entry windows

Without propulsion, the orbit plane is almost fixed in an inertial reference system (except for non-central-force perturbations), whereas Earth rotation carries the launch pad around. Launch trajectory is basically a two-minute vertical impulse to go over the atmosphere, followed by a horizontal kick along the orbit; thence, for the launcher to place a spacecraft in a given orbit, the kick must be done precisely at the moment the rotation-entrained launch-site is in the orbit plane envisaged; otherwise, the spacecraft would be placed in a different orbit plane, and additional propellant would be required (a lateral delta-v) to change from one orbit to the other.

Similarly, entry trajectory can be seen as a sudden vertical plunge on the landing spot chosen. Landing site and current orbit plane coincide twice per day. However, contrary to the launch trajectory (where we has plenty of time to slightly modify the spacecraft orbit position for a rendezvous if desired), entry is a one shot event, with little time to change the entry trajectory one initiated, so that, besides being in an orbit passing over the intended landing spot, the spacecraft must be in the correct orbit position knowing that, after the deorbit manoeuvre, it will cover some 10 000 km on the ground track while falling from 300 km to 100 km (in a half an hour run), plus another 2000 km to land if a capsule, or 6000 km to land if a Shuttle, to touch down. The acceptable dispersion in landing spot may be several-hundred-kilometres wide (for capsules, or can be somehow compensated in shuttle gliding), so that two consecutive orbits may be valid, one to either side of the intended landing spot, because the maximum ground separation between consecutive orbits (the rotation of the Earth in one LEO period) is (1.5 h)·(40 000 km)/(24 h)=2500 km.

Launch site and prescribed orbit plane coincide twice per day. The duration of the window varies with season due to the tilt of Earth's rotation axis. Of course, local weather conditions pose additional constraints on top of these launch-window restrictions (i.e. all schedules are always understood 'weather permitting'). For instance, to reach the ISS, the Shuttle must take off within about five minutes of the moment when Earth's rotation carries the launch pad into the plane of the station's orbit.

Type of launchers and propellants used

A typical launcher classification by size (really payload capacity to LEO) may be:

- **Super-heavy**, if $m_{\text{pay}}>50,000$ kg to LEO: Saturn V (120 t to LEO, designed for Apollo), Energia (90 t to LEO, used to launch Buran), Falcon Heavy (53 t to LEO, under development). The USA future Space Launch System (SLS) aims at 130 t to LEO.
- **Heavy**, if $m_{\text{pay}}=10...50$ t to LEO: Long March (25 t to LEO), Space Shuttle (24 t to LEO), Delta IV (23 t to LEO), Titan IV (22 t to LEO), Proton (22 t to LEO), Ariane 5 (21 t to LEO), H-IIB (20 t to LEO),
• Medium, if $m_{pay} = 1..10$ t to LEO: Falcon 9, H-IIA, Atlas V, Soyuz U, PSLV (India)...
• Small, if $m_{pay} < 1.5$ t to LEO: Vega...

For comparison reasons, Table 3 presents large LOX/LH2 engines of American, European, Japanese, and Russian manufacture, together with two LOX/kerosene engine (one American and one Russian, also used in the US-Atlas rocket since the 1990s). Typically all cryogenic engines have a turbine and a pump for each of the two propellants; only the RD-180 has one single preburner which delivers the necessary driving gas to a one single stage turbine which again provides for the thrust requirements of the two propellant pumps, a single stage oxygen pump and a two stage fuel pump.

<table>
<thead>
<tr>
<th>Propellants</th>
<th>Vulcain 2</th>
<th>SSME</th>
<th>LE-7A</th>
<th>RD-0120</th>
<th>F-1</th>
<th>RD-180</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ariane 5 (ESA)</td>
<td>LOX/LH2</td>
<td>LOX/LH2</td>
<td>LOX/LH2</td>
<td>LOX/LH2</td>
<td>LOX/RP1</td>
<td>LOX/RP1</td>
</tr>
<tr>
<td>Propulsive-machinery</td>
<td>LOX/LH2</td>
<td>LOX/LH2</td>
<td>LOX/LH2</td>
<td>LOX/LH2</td>
<td>LOX/RP1</td>
<td>LOX/RP1</td>
</tr>
<tr>
<td>$T_{GG}$ [K]</td>
<td>875</td>
<td>900</td>
<td>810</td>
<td>846</td>
<td>816</td>
<td>820</td>
</tr>
<tr>
<td>$p_{GG}$ [MPa]</td>
<td>10</td>
<td>36</td>
<td>21</td>
<td>42</td>
<td>?</td>
<td>55</td>
</tr>
<tr>
<td>$m_{GG}$ [kg/s]</td>
<td>9.7</td>
<td>80</td>
<td>53</td>
<td>78.6</td>
<td>?</td>
<td>887</td>
</tr>
<tr>
<td>$F_{turb}$ [MW]</td>
<td>14</td>
<td>56</td>
<td>19</td>
<td>62</td>
<td>44</td>
<td>93.5</td>
</tr>
<tr>
<td>$p_c$ [MPa]</td>
<td>12</td>
<td>21</td>
<td>13</td>
<td>22</td>
<td>7</td>
<td>26</td>
</tr>
<tr>
<td>$F$ [MN]</td>
<td>1</td>
<td>2</td>
<td>0.9</td>
<td>1.8</td>
<td>6.7</td>
<td>4</td>
</tr>
<tr>
<td>$\Delta t_{burn}$ [s]</td>
<td>600</td>
<td>480</td>
<td>350</td>
<td>600</td>
<td>160</td>
<td>250</td>
</tr>
</tbody>
</table>

**Ariane 5 engines**

**Ariane 5** is a heavy launcher by ESA; version ECA (Évolution Cryotechnique type A) is able to place 10.5 t in GEO, whereas version ES (Evolution Storable) can put 21 t in LEO (used to launch the Automated Transfer Vehicle, ATV). It consists of two solid boosters, and two liquid stages, being 50 m tall and 5.4 m in diameter (solid boosters apart). Lift-off mass is 777 t. Besides, there are 400 N hydrazine attitude control rockets.

**Solid boosters**

Each of the two EAP solid rocket boosters (Étages d’Accélération à Poudre) is 30 m tall, providing a thrust of $F = 6500$ kN during 129 s, with 238 t of propellant, AP-Al-HTPB 68%-18%-14% with $M = 0.0274$ kg/mol, $\rho = 1770$ kg/m$^3$, burning rate $v_r = 9$ mm/s, $T_{max} = 3330$ K, $\gamma = 1.14$, and a specific impulse of $I_{sp} = 275$ s ($v_e = 2750$ m/s). Solid boosters detach after 130 s, with ocean splashdown. Derived from the EAP rocket is the P80 used as first stage in Vega launcher; the booster casing is made of filament wound graphite epoxy, much lighter than the current stainless steel casing.

**First stage (Vulcain 2)**

The first stage of Ariane 5 ECA uses one Vulcain-2 cryogenic (LH2/LOX) rocket engine burning during 650 s of operation. At lift-off it provides 900 kN thrust (8% of the total lift-off thrust, the rest being provided by the solid boosters); vacuum thrust is 1340 kN. Ariane 1st stage is 30 m long, and carries 133 t of LOX.
(120 m³) and 26 t of LH₂ (390 m³), with a dry mass of 11 t (total mass at lift-off is 170 t); after completing its propulsive mission, the empty stage is commanded to re-enter the atmosphere for an ocean splashdown.

Vulcain-2 is 3.4 m tall and has 2.1 m rim diameter, with 1800 kg dry mass (maximum thrust/weighs ratio is \( F/W = 75 \)). Turbopumps, driven by gases from a gas generator fed from a small bleeding of propellants, feed the main combustion chamber, operating at 11.5 MPa and 3500 K, with \( I_{sp} = 460 \text{ s} \) \( (v_e = 4500 \text{ m/s under vacuum}) \). The oxygen turbopump rotates at 13 000 rpm with a power of 4 MW and a flow rate of 195 kg/s, while the hydrogen turbopump rotates at 35 000 rpm with 13 MW of power and a flow rate of 41.2 kg/s. Vulcain-2 features regenerative cooling through a tube wall design in the nozzle, and gas film-cooling for the lower part of the nozzle, where exhaust gas from the turbopumps is injected (Fig. 11). The main contractor for the Vulcain engines is Snecma Moteurs (France), which also provides the liquid hydrogen turbopump. The liquid oxygen turbopump is responsibility of Avio (Italy), and the gas turbines that power the turbopumps and the nozzle are developed by Volvo (Sweden).

![Fig. 11. Ariane 5 (ECA version): a) General launch configuration; b) Vulcain-2 engine sketch; c) Vulcain-2 between the two solid boosters on the launch pad; c) Vulcain-2 ground test (Snecma).](image)

**Second stage (HM7-B)**

The second stage of Ariane 5 ECA utilizes a HM7-B rocket, burning LH₂/LOX for 960 s and providing \( F = 65 \text{ kN} \) with \( I_{sp} = 446 \text{ s} \); rocket mass is 2100 kg, and the total mass of propellants is 14 t.

The second stage of Ariane 5 ES utilizes a Aestus rocket, burning N₂O₄/MMH for 1100 s and providing \( F = 27 \text{ kN} \) with \( I_{sp} = 324 \text{ s} \).

**Soyuz 2 engines**

Soyuz (‘union’ in Russian) may refer to a rocket family (Fig. 12) or a spacecraft. Soyuz 2 rocket is a new member of the R-7 rocket family, which has been the most used worldwide. It is a 2- or 3-stage rocket, 46 m high and 3 m in diameter, with a mass of 305 t, able to place payloads up to 7.8 t into LEO.
Boosters
Four liquid-fuelled RD-107A boosters, burning RP1/LOX for 120 s and then discarded, each with a thrust of $F=840$ kN at sea-level (1020 kN in vacuum), with exhaust speed $v_e=3100$ m/s ($I_{sp}=300$ s at sea level). The 4 boosters are arranged around the central core and are tapered cylinders with the oxidizer tank (LOX) in the tapered portion and the kerosene tank in the cylindrical portion. These spark-ignition engines are fed by a turbopump driven by gases generated by the catalytic decomposition of H2/O2 in a gas generator. Each RD-107A has four combustion chambers and nozzles. Liquid nitrogen is used for pressurization of the propellant tanks.

Core stage
One RD-118A engine, burning RP1/LOX for 300 s, with a thrust of $F=800$ kN at sea-level (1000 kN in vacuum), with exhaust speed $v_e=3100$ m/s ($I_{sp}=300$ s at sea level). This second stage is similar in construction to the booster stage, and is shaped to accommodate the boosters, and has four vernier thrusters for three-axis flight control.

Upper stage (optional)
This is a re-startable S5.92 (Fregat engine), burning N2O4/UDMH for 900 s, with $F=20$ kN, speed $v_e=3300$ m/s ($I_{sp}=340$ s under vacuum), and holding the fairing with the adapters and payload.

The third stage of the Soyuz 2-1b is powered by the RD-0124 engine burning RP1/LOX. The RD-0124 engine is a staged combustion engine powered by a multi-stage turbopump spun by gas from combustion of the main propellants in a gas generator. These oxygen rich combustion gases are recovered to feed the four main combustion chambers where kerosene coming from the regenerative cooling circuit is injected. LOX and kerosene tanks are pressurized by the heating and evaporation of helium coming from storage vessels located in the LOX tank. The four nozzles of the main engines are gimbaled along two perpendicular planes to provide attitude control.
Vega engines

Vega is an ISA-ESA launcher (65% I, 13% F, ES, BE, NE, SW & SW) to put small payloads (around 1500 kg, in the range 300..2000 kg) in LEO, at some 700 km altitude polar orbit, for scientific and Earth observation missions. Vega is a single-body launcher (no strap-on boosters), made of CFRP, with three solid rocket stages, the P80 first stage, the Zefiro 23 second stage, the Zefiro 9 third stage, all with carbon-carbon nozzles, and a re-ignitable liquid rocket upper module called AVUM using N_2O_4/UDMH and supporting the payload fairing.

The technology developed for the P80 program will also be used for future Ariane developments. The Zefiro 9 third stage is 3.7 m long, 1.9 m in diameter, with 10 500 kg APCP in a 390 kg case, and delivers 250 kN thrust for 110 s, with maximum chamber pressure of 7.5 MPa, nozzle diameter of 0.16 m at the throat and 9.7 m at the exit, and specific impulse \( I_{sp} = \frac{v_e}{g_0} = 2940 \text{ m/s} \).

Long March engines

Long March rockets (LM, or CZ for Chang Zheng, meaning Long March in Chinese pinyin) is a family of Chinese rockets.

- Long March 1's 1st and 2nd stage used nitric acid and UDMH propellants, and its upper stage used a spin-stabilized solid rocket engine.
- Long March 2, Long March 3, Long March 4; the main stages and associated liquid rocket boosters used N_2O_4/UDMH. The upper stages (third stage) of Long March 3 rockets (YF-73 and YF-75) used LH2/LOX.
- The new generation of Long March rocket family, Long March 5, and its derivations (Long March 6...) use RP1/LOX in the core stage and liquid boosters, and LH2/LOX in upper stages.

THE SPACE SHUTTLE

The Space Shuttle (officially the Space Transportation System, STS) has been the only reusable spacecraft up to now; it was in service from 1981 to 2011, and was upgraded regularly, particularly on avionics (e.g. it started using five 1970's computers with 424 kilobytes of magnetic core memory each, loading software from magnetic tape cartridges; without hard disks). The Shuttle was one of the earliest craft to use a computerized fly-by-wire digital flight control system.

In the 1980s and 1990s, many Shuttle flights involved scientific missions on the NASA/ESA Spacelab, or launching various types of satellites and science probes; by the late 1990s and 2000s the focus shifted to servicing the space station (ISS). Major drawbacks have been a huge underestimation of operation and maintenance costs (aggravated by using crewed vehicles for freight), and many technical flows on thermal issues (pieces of the brittle external-tank-insulation detaching during launch, refractory tiles fragmenting or coming unstuck, and brittle O-rings in solid rocket joints).

The STS consists on the Orbiter Vehicle (OV, known as the Shuttle, holding several rocket engines), a pair of recoverable solid rocket boosters (SRB), and the expendable external tank (ET). For lift-off (from KSC, Florida), the two SRB and the three Shuttle main engines (SME) were used at a time (Fig. 13), the latter
fed with LH2/LOX from the ET, the SRB burning for 2 min and the SME for 8.5 min. At launch, the mass and thrust shares are:

- Total lift-off mass $2.05 \cdot 10^6$ kg: 58% the two SRB, 37% the ET, and 5% the Orbiter.
- Total lift-off thrust $30.2 \cdot 10^6$ N: 83% the two SRB, 17% the three SME (fed from the ET).

![Image of Space Shuttle launching](image1)
![Image of Space Shuttle](image2)

In addition to the two SRB and the three SME, the orbiter contains two orbital manoeuvre system (OMS) engines, providing 27 kN each, and 44 reaction control system (RCS) thrusters, to carry out docking manoeuvres in orbit, and control attitude during descent. The OMS engines use monomethyl hydrazine as the fuel and nitrogen tetroxide as the oxidizer (MMH/N₂O₄), which are hypergolic liquids (ignite on contact), 500 kg in total (for a total delta-v of $\Delta v=300$ m/s), fed by high-pressure helium (there is also a tank that provides pressurized nitrogen to operate the engine valves). The minimum duration of an OMS engine firing is 2 s.

RCS thrusters were grouped in the nose of the vehicle and on each of the two aft OMS-pods (no nozzles interrupted the heat shield on the underside of the craft, instead, the nose RCS nozzles which control positive pitch were mounted on the side of the vehicle, and were canted downward, as seen in Fig. 14b; the downward-facing negative pitch thrusters were located in the OMS pods mounted in the tail/after-body).
The Space Shuttle has three redundant auxiliary power units (APU), powered by hydrazine fuel. They function during ascent, reentry, and landing. During ascent, the APUs provide hydraulic power for gimbaling of Shuttle's main engines and control surfaces. During descent and landing, they powered the control surfaces and brakes.

The orbiter re-enters by firing the OMS to de-orbit (Fig. 14c), and glides to a runway landing on KSC, or at Edwards Air Force Base in California (EAFB, and using one of the two NASA modified B747 aircraft to carry it back to KSC); weather at EAFB is more stable than at KSC, and almost half of all landings were there (and one in New Mexico), in spite of the extra cost (and one week delay for next launch).

Five Shuttle vehicles were built (Columbia, Challenger, Discovery, Atlantis, and Endeavour, besides Enterprise, a unit built just for approach and landing tests), with two of them lost in accidents: Challenger in 1986 during launch (by malfunction of the SRB), and Columbia in 2003 during reentry (by damage on the orbiter wing due to a foam piece detachment from the ET at launch). From the first flight of STS-1 with Columbia in 1981 to the last flight of STS-135 with Atlantis, 135 launchings were performed, and 355 people have flown.

Lift-off and ascent
Recall that, to put a body in orbit, much more horizontal than vertical acceleration is required; after lift-off from KSC, the ascent is progressively tilted North-East, although not visually obvious, since it is soon too small to be seen (e.g. a x=7 km lateral displacement at z=10 km altitude means a zenith deviation of arctan(7/10)=35º unnoticeable in a sky view without a vertical reference). Notice also that vertical speed decreases after SRB end at TO+125 s (z=60 km), and becomes negative at some later time, because the vertical thrust is less than the weight, but the loss of mass soon overcomes it.

Orbit inclination is chosen depending on the mission, from \(i=28.5^\circ\) used for GEO satellites (the minimum from KSC), to \(i=52^\circ\) used for ISS rendezvous.

Nominal mass and thrust values for the STS at launch are:
- Mass: 2030 t total (1140 t for the solid boosters, 760 t for the ET (26.5 t empty), and 110 t for the loaded Orbiter).
- Thrust: 30 MN (compare with 20 MN of launch weight). Each of the SRB yields 12.5 MN, and each of the three SME yields 1.8 MN. Each of the two second-stage rockets (OME) yield 27 kN.

A time-plot of altitude, speed, and horizontal displacement during Shuttle ascent is presented in Fig. 15 (until SRB separation 125 s after take-off, TO); several video recordings can be found on the web. More details are given in Table 4 (a summary of typical launch events was given in Table 2, and a generic evolution during both launching and reentry phases was presented in Fig. 7, above).
Table 4. **Space Shuttle timeline** during lift-off and ascent.

<table>
<thead>
<tr>
<th>Time (TO)</th>
<th>Event Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6 hours</td>
<td>Start of cryogenic propellants filling</td>
</tr>
<tr>
<td>-30 s</td>
<td>Control transfer</td>
</tr>
<tr>
<td>-7 s</td>
<td>Main engines start</td>
</tr>
<tr>
<td>(take-off)</td>
<td>Solid boosters start</td>
</tr>
<tr>
<td>+5 s</td>
<td>z=75 m, v=30 m/s Tower cleared</td>
</tr>
</tbody>
</table>

The filling continues until just before lift-off. The crew enters the Orbiter about 2.5 h before lift-off (with helmets on). At TO-7 minutes the crew access platform is removed.

Ground crew manually sets each SRB-lock pin to enable, and whole control is transferred to the Shuttle's on-board computers. At TO-15 s, a pool underneath is flooding with 1300 m$^3$ of water for acoustic and rocket plume damping (even so, the sound level hurts the ear 6 km around).

The 3 SME are sequentially ignited with 0.12 s interval (can be stopped), burning 1300 kg/s with 4600 m/s at exhaust; the 5.3 MN thrust is transmitted to launch pad with noticeable deformations (at TO-2 s with 100 % power, the ET-tip shifts some 0.65 m).

The 2 SRB are ignited at a time (cannot be stopped), burning 9300 kg/s with 2700 m/s at exhaust, producing a 25 MN total thrust. The Orbiter computers send the signal that triggers a condenser that ignites a pyrotechnic device that sets fire on the SRB propellant. At the same time, 8 pyrotechnic nuts release the SRBs from the pad, and umbilicals are retracted (LOX line, LH2 line, and GH2 vent line at inter-tank). At TO+0.1 s there is $p_c=0.3$ MPa in the SRB chamber and a protective nozzle-throat-plug bursts; at TO+0.5 s is $p_c=6$ MPa (nominal chamber pressure).

After the Shuttle clears the tower, the STS rolls right 180º to have the Orbiter due East, and shifts towards the Orbiter (pitch-up), to create a negative angle of attack that presses the Orbiter to the ET, decreasing structural loads and global air drag. This head-down wings-level also helps radio communications and a possible abort manoeuvre (the pilot sees the horizon). Mission control shifts from KSC to JSC. The thrust takes an East component for low-inclination orbits, or North-East for higher inclinations*. At TO+4 s SME power rises from 100 % to 105 %
until TO+45 s, when it is throttle down from 105 % to 70 % just after crossing drag divergence around $M=0.9$ at about $z=5$ km.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Conditions</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>TO+60 s</td>
<td>$z=10$ km, $v=500$ m/s</td>
<td>$M=1.8$. Maximum dynamic pressure ($\Delta p=35$ kPa); SMEs were throttled down to limit dynamic pressure for structural strength. Condensation clouds (Fig. 13b) may form during transonic flight.</td>
</tr>
<tr>
<td>TO+125 s</td>
<td>$z=45$ km, $v=1.6$ km/s**</td>
<td>$M=4.1$ (almost 45 km down range KSC). Solid propellant runs out, and pyrotechnic fasteners release the SRBs (small separation rockets pushed them laterally away from the vehicle). The SRB go up for a while, Fig. 13d, and fall, with a parachute softening the splash in the ocean (some 250 km East of Florida), to be recovered and reused.</td>
</tr>
<tr>
<td>TO+250 s</td>
<td>$z=100$ km, $v=2.7$ km/s</td>
<td>$M=8.1$ (250 km down range). The Shuttle cannot return to KSC and the abort landing site shifts to Europe (e.g. to Zaragoza, or Dakar).</td>
</tr>
<tr>
<td>TO+350 s</td>
<td>$z=110$ km, $v=4.1$ km/s</td>
<td>$M=12$ (550 km down range). Direct radio link to ground is lost, and the Shuttle rolls to place the Orbiter atop the external tank to allow satellite communications via TDRSS (Tracking and Data Relay Satellite System).</td>
</tr>
<tr>
<td>TO+510 s</td>
<td>$z=110$ km, $v=7.8$ km/s</td>
<td>No thrust. Weightlessness starts, still in suborbital flight (to get rid of the ET and to abort in case of OMS malfunction). Pyrotechnic fasteners release the ET, which re-enters and burns up in the atmosphere: firstly the foam burns away, then the heat causes a pressure build-up in the remaining liquid oxygen and hydrogen until the tank explodes (some cryogenics must be left, since the SME are ruined if working dry), and finally some fragments fall on the Indian Ocean.</td>
</tr>
<tr>
<td>TO+600 s</td>
<td>$z=120$ km, $v=7.8$ km/s</td>
<td>About 100 s after MECO, the two OMS rockets are fired to raise the attainable apogee, although this firing may be skipped if previous thrust is enough, or may be initiated before MECO. Each OMS fires at 9 kg/s ($F=27$ kN, $I_{sp}=315$ s) for about 150 s in one or several steps. On later flights to the ISS, the OMS was started before ET separation.</td>
</tr>
<tr>
<td>TO+2500 s</td>
<td>$z&gt;300$ km, $v=7.7$ km/s</td>
<td>The two OMS rockets are fired to get a stable circular orbit, and then cut off, until needed for a further orbit manoeuvre like rendezvous with the ISS (RCS rockets are used for docking), or for reentry (where they are retro-fired for about 3 minutes to deorbit). About TO+1.5 hours the bay doors open (needed for cooling) and remain open until reentry time around 10 days later.</td>
</tr>
</tbody>
</table>

*As the ISS orbits at an inclination of 51.6º, the Shuttle has to be launched (after tower clearance) at 51.7+3=55º (i.e. North-East), and at the time when ISS orbit passes by, in order to rendezvous without major orbit changes; the 3º is to compensate for the 410 m/s Earth entrainment at KSC.

**That speed value and subsequent values in the Table are not relative to ground but inertial, i.e. adding the Earth rotation entrainment that is 410 m/s due East at KSC (28.6ºN); i.e. with a 45º path angle at that point, relative speed is about 1600−410·cos45º=1310 m/s.
Exercise 5. Consider the following data for the Space Shuttle at launch: lift-off mass $2 \times 10^6$ kg, initial thrust 30 MN, and initial mass flow rate 10 000 kg/s. Estimate:

a) The acceleration at take-off (TO), and 100 s after.

b) The altitude and elapsed time to achieve Mach $M=1$.

Sol.:

a) The acceleration at take-off time (TO) is obtained from the force balance just after lift-off: $ma=F-mg$; i.e. $a_0=F/m_0-g=30/2-9.8=5$ m/s$^2$, corresponding to a g-force of 1.5 $g$.

As a curiosity, we may find the exit speed of gases in the rockets, $v_e=pF/m_\dot m=30\times10^6/(10\times10^3)=3000$ m/s ($I_{sp}=310$ s). In fact, it is an hybrid launcher that lifts off with 5 rockets at a time: two SRB provide 25 MN burning 8.3 t/s with $I_{sp}=260$ s, and three SME providing 5 MN burning 1.7 t/s with $I_{sp}=450$ s.

The acceleration at TO+100 s is not easy to find because of the air-drag contribution, and gravity-turn effect. Neglecting air-drag and assuming vertical motion, the force balance is again $ma=F-mg$, but now the mass is $m=m_0-m_\dot m=2\times10^6-10\times10^3\cdot100=1\times10^6$ kg (i.e. half the lift-off mass is spent in the first 100 s!); i.e. $a_{100}=F/m-g=30/1-9.8=20$ m/s$^2$, corresponding to a g-force of 3 $g$ (2.5 $g$ from actual data). In reality, thrust is not constant during the 125 s that the SRB burn: most of the time its power is at 105 % of the nominal value given, and during 15 s after TO+30 s they are throttles to 70 % of nominal power.

b) An atmosphere model would be needed to know the temperature profile $T(z)$ for the speed of sound ($c=\sqrt{\gamma RT}$, with $\gamma=1.4$ and $R=287$ J/(kg·K)), and the density profile $\rho(z)$ to compute air drag with $D=\frac{1}{2}\rho v^2cDA$, besides the frontal area (might be estimated as $A=100$ m$^2$), and the air-drag coefficient for that shape (four cylinders with attachments: ET, OV, and two SRB), which may be assumed about $c_{D0}=0.5$, although it is not a constant but a function of airspeed, angle of attack, yaw angle and a few other variables (and has a pronounced peak around $M=0.9$).

But we first get an estimate assuming constant temperature with $c=340$ m/s, and negligible air drag, and then the force balance admits analytical integration to yield the trajectory:

$$m\frac{d^2z}{dt^2}=F-mg$$

$$\frac{m}{m_0-m_\dot m}z(t)=-\frac{1}{2}gt^2+\frac{F}{m}\left(1-\ln\frac{m_0-m_\dot m}{m_0}\right)t+\frac{Fm_0}{m^2}\ln\frac{m_0-m_\dot m}{m_0}$$

which can further be expanded in series for $t\to0$ to check that with constant mass and thrust, the solution would be $z(t)=-\frac{1}{2}gt^2+\frac{1}{2}(F/m_0)t^2$. Solving the condition $M=1$ ($dz/dt=c$) for the unknown time one gets $t_1=c/(F/m_0-g)=70$ s, or $t_1=47$ s with the mass-decreasing solution above stated. The corresponding altitude is $z_1=12$ km with the constant-mass model, and $z_1=7.1$ km with the mass-decreasing model. From real trajectory data, $M=1$ is obtained at TO+44 s, $z=7.2$ km, $v=323$ m/s, where the g-force is 1.7 $g$, and the vehicle has already 3.3 km of lateral ground displacement.

Had we used a more detailed model for the trajectory, with appropriate density and temperature profiles, and $c_D$ as a function of Reynolds number, the result would had been nearly the same because air drag is a minor force, as we could have guessed from the maximum dynamic pressure quoted in Table 4, since to a $\Delta p=35$ kPa=$\frac{1}{2}\rho v^2$, corresponds a maximum drag $D=\frac{1}{2}\rho v^2cDA=35\times10^3\cdot0.5\cdot100=1.75$ MN, much smaller than the thrust $F=30$ MN.
**Challenger disaster**

Launch of STS-51 on 1986-01-28 was catastrophic, leading to the death of its seven crew members. Disintegration of the vehicle began after the lower of the three O-ring-seals in its right SRB lost tightness and let some pressurized hot smoke to escape at intervals (the unusual high shear winds amplified the hot gas escape by mechanical bending and thermal melting of the lining). The O-ring material loses elasticity below 0 °C, and there was freezing weather (−1 °C) in the early morning before lift-off (11:38 local time); lift-off had been delayed several days because of bad weather, the last time 1 h before to get rid of ice on the pad; previously, the coldest launch had been at 12 °C (some 30 % of Shuttle launch delays were caused by bad weather). Each SRB was made of seven sections, six of which permanently joined in pairs at the factory; the four resulting segments were then assembled at KSC with three field joints sealed with two rubber O-rings each (three O-rings after this accident); rocket thrust added compression to the O-rings.

At TO+59 s (z=15 km) a jet flame was clearly seen to escape and impinge onto the external tank, which ruptured soon after and destabilised the STS; the Orbiter broke up by aerodynamic forces at TO+73 s, \( M=2 \). Massive burning of the liberated cryogenics took place, and the solid boosters were exploded by ground command for launch-pad safety. Later review of launch film showed that already at TO+0.7 s, strong puffs of dark grey smoke were emitted from the right-hand SRB near the aft strut that attaches the booster to the ET. If the escaping jet had not pointed to the tank, a loss of thrust from that booster would have triggered an abort manoeuvre for the orbiter to detach and try landing back at the site. The Space Shuttle fleet was grounded for nearly three years.

**Columbia disaster** (STS-107), although happened during reentry at about 70 km altitude, it originated during launch, at about TO+82 s and 20 km altitude, when a piece of about 3 kg of polyurethane foam from the ET-insulation detached and, blown by a relative air-speed of some 840 m/s (\( M=2.5 \)), made a hole about 0.2 m in size) in the thermal protection panels (carbon fibre carbon reinforced shaped panels) at the leading edge of the left wing.

**Engines: SRB, SME+ET, OMS, RCS**

The Space Shuttle had two solid rocket boosters (SRB), three Shuttle main engines (SME), two orbital manoeuvre systems (OMS), and 44 small thrusters (reaction control system, RCS).

**SRB**

The Shuttle solid boosters (SRB), made by Morton Thiokol, are the largest solid rockets ever built and flown, each 3.7 m in diameter, 45.5 m long, with 500 t of propellant (APCP: 70 % AP, 16% Al, 12.5 % PBAN1.5 % ECTV), with 90 t empty mass. They burn only during the first 125 s (500/125=4 t/s each), reaching an altitude of \( z=45 \) km (not vertically but with another 45 km of lateral displacement), and are then detached (but continue raising in a suborbital flight up to 65 km of apogee before falling some 230 km off-range in the Atlantic, some 300 s after separation. Each SRB has a nominal thrust of \( F=12.5 \) MN, with exit speed of \( v_e=2500 \) m/s (\( I_{sp}=260 \) s) at sea level. Actual dispersions in thrust direction and intensity, although <5 %, requires some trimming for precise manoeuvring.

**SME with ET**

Spacecraft propulsion
The Shuttle main engines (SME) are RS-25 rockets, the first reusable large cryogenic engines (made by Rocketdyne; first tests in 1977, first flight in 1981). Each one has a sea-level thrust of 1.7 MN (2.3 MN under vacuum). They feed from separate reservoirs in the external tank (ET) via umbilicals. NASA intends to use the RS-25 engine for its future Space Launch System (SLS).

The external tank (ET) holds the cryogenic propellants used in the three SME; it is made of aluminium 12 mm thick (with some stiffeners), has 8.4 m in diameter, is 47 m long (30 m for the 118 t of LH2 at 20 K and 230 kPa, 12 m for the 764 t of LOX at 90 K and 250 kPa, and the rest for ancillaries), 26.5 t empty, and 760 t fully loaded. The thermal protection system of the ET insulates the cryogenic liquids inside from the outside environment, and bears the aerodynamic heating during ascent; it is made of polyurethane foam (mostly sprayed on site over the 12 mm aluminium structure; $\rho=40\ldots45$ kg/m$^3$, $k=0.2$ W/(m·K)), about 25 mm thick, with some special pieces at joints, and an external water-proof layer to avoid condensation of air moisture (white in the first two ET flown); a total of 2200 kg foam insulation.

Large cryogenic tanks are always kept with little overpressure to minimize wall thickness, but in this case, a few minutes before lift-off, helium gas is added to guarantee there is no cavitation on pump suction; i.e. at TO-3 minutes, 150 kPa of He is added to the 100 kPa of the LOX vapours to have the 250 kPa on top (at the LOX bottom there is another 150 kPa of hydrostatic contribution); notice that if pure oxygen were at 250 kPa in two-phase equilibrium, its temperature would be 100 K instead of 90 K).

The LOX tank is on top to have an advanced centre of mass for aerodynamic stability. Both feeding lines are 0.43 m in diameter, supplying 210 kg/s of LH2 (3 m$^3$/s, at 21 m/s), and 1300 kg/s of LOX (1.1 m$^3$/s, at 7.6 m/s). Combustion of LH2 and LOX is performed in two stages. In a first chamber, some 75 % of LH2 (the whole flow rate except the part used for cooling) is burned with a small amount LOX (some 10 %) in an extremely rich combustion, started by spark igniters (propellants being fed by tank pressure). The resulting hot gas (mainly H$_2$ at near 1000 K) is used to power the direct drive (no reduction gear) turbopump turbine that pumps the main LOX flow (and the coolant fuel) into the main combustion chamber; a part of this H$_2$ flow is used to pressurize the fuel in the external tank (ET).

Fuel and oxidiser enter the main combustion chamber (MCC) mostly in the gas phase: the hydrogen is vaporized in the preburner and in the cooling circuit, and the oxygen is partially vaporized by the hydrogen in a co-flowing heat exchanger arrangement ahead of the injectors, and the rest arrives in liquid state, is atomized, and evaporates shortly after being injected. The burnt gases in the MCC attain 3250 K and 20 MPa. The structural wall is made of Inconel-718, with a maximum service temperature of 930 K, but it is forced cooling by the cryogenic fuel outside (besides using some colder boundary layer flow inside; see Rocket cooling, above). The Inconel shell is lined with a copper-silver-zirconium alloy called NARloy-Z (developed specifically for the RS-25 in the 1970s), with some 390 channels machined into the liner wall to let the cryogenic hydrogen to flow through and provide cooling of the wall.

At lift-off only 65 % of the nominal power is developed, to avoid flow separation in the over-expanded nozzle, and each rocket is fed with $\dot{m}_{H_2}=70$ kg/s and $\dot{m}_{O_2}=450$ kg/s (notice some 25 % H$_2$ excess over stoichiometry), with the main combustion chamber working at 20 MPa and 3250 K. The nozzle has an area...
ratio $\varepsilon = A_e / A_t = 77$, with throat diameter $D_t = 0.262$ m and exit diameter $D_e = 2.30$ m (at a distance of 3 m). Each empty rocket has 3500 kg.

The exhaust gas has an average molar mass $M = 0.014$ kg/mol (mainly H$_2$O, some H$_2$ excess, and traces of H, and OH), with a mean thermal capacity (accounting for the large temperature range) $c_p = 50$ J/(mol·K) = 3500 J/(kg·K) ($\gamma = 1.20$). It expands from $p_0 = 20$ MPa to $p_e = 35$ kPa, with an exit adiabatic temperature $T_e = T_0 (p_e / p_0)^{(\gamma - 1)/\gamma} = 3250 \cdot (0.035/20)^{2/1.2} = 1130$ K, and exit speed $v_e = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2 \cdot 3500 \cdot (3250 - 1130)} = 3850$ m/s, with a thrust $F = \dot{m}v_e + A_e(p_e - p_0) = (70+450) \cdot 3850 + (\pi \cdot 1.26^2/4) \cdot (35-100) \cdot 10^3 = 2.0 \cdot 10^6 - 0.27 \cdot 10^6 = 1.73$ MN each (5.2 MN among the three), and specific impulse $I_{sp} = F/(\dot{m} v_e g) = 1.73 \cdot 10^6 / ((70+450) \cdot 9.8) = 340$ s. Rocket thrust to weight ratio is $F/W = 1730/35 = 50$.

The SME can be throttled over a range of 65..109% in 1% increments by opening the throttling valve governing output of the pre-burning chamber, increasing the speed of the main pumps, flow rates, and chamber pressure. The SME can be gimballed $\pm 12^\circ$ in two planes by hydraulic actuators for vehicle pitch, yaw, and roll control. At sea level (take-off, TO), the engine is throttling to 65% of nominal power to avoid flow separation in the nozzle, but a TO+4 s full power is applied until at TO+45 s throttling goes back to 65% to avoid excessive aerodynamic forces (dynamic pressure rises to 35 kPa). At TO+70 s nozzle flow is no longer over-expanded and the engine is rated to full power. Engine thrust increases from 1.7 MN at sea level to 2.3 MN under vacuum (exit speed increases from 3400 m/s to 4400 m/s).

To appreciate the huge power releases during lift-off, knowing the lower heating value of H$_2$, $h_{LHV} = 120$ MJ/kg, and multiplying for the mass-flow-rate that reacts (only 180 kg/s from the 210 kg/s of LH2 react, the rest is used for dilution), $120 \cdot 10^6 \cdot 180 = 22$ GW are released (converted from internal chemical energy to internal thermal energy), a minor part of which is converted to mechanical propulsion power (thrust times advance speed), the rest being dissipated in the exhaust stream. For comparison, nuclear power plants generate 1 GW of electricity from the conversion of 3 GW of internal nuclear energy to internal thermal energy. During Shuttle launch, besides the 22 GW released in this liquid rockets (the three SME), another 11 GW are released in the two solid rockets (SRB; 2 times 4 t/s times $h_{LHV} = 1.4$ MJ/kg for APCP).

**OMS and RCS**

The Shuttle has two orbital manoeuvring engines (OMS) located in pods on the aft section of the orbiter (one on either side of the tail), and are used to place the Shuttle into final orbit, to change its orbit, and to deorbit for reentry. The reaction control system (RCS) in the Shuttle contains a total of 38 primary thrusters (each of 3.9 kN) and 6 vernier thrusters (each of 0.10 kN); 14 primary thrusters and two vernier thruster are located at the forward end of the Orbiter, and 12 primary thrusters and two vernier thrusters are housed in each of the two OMS/RCS pods.

Both OMS and RCS use the same propellants: MMH/N$_2$O$_4$ stored in separate domed-cylindrical titanium tanks 1.2 m in diameter, each pressurized by helium that pushes the fluids through the fuel lines (without pumps), with a full propellant load of 8200 kg of MMH and 13 500 kg of N$_2$O$_4$, capable of a total delta-$v$ of $\Delta v = 300$ m/s onto a fully loaded Orbiter ($m = 110$ t). Specific impulse is $I_{sp} = 315$ s ($v_e = 3100$ m/s) under vacuum. A pod-cross-feed line allows the propellants in each pod to be used to operate any engine.
The OMS engines are derivatives of Aerojet's AJ-10 series of upper stage rockets used since the Vanguard project (1957). Each has an empty mass of 120 kg (with tanks), a thrust of 27 kN, and can be gimballed to provide pitch and yaw control. The nominal flow rate to each engine is 3.6 kg/s of MMH and 6 kg/s of N₂O₄. Pressure in the combustion chamber is 0.9 MPa. Nozzle area ratio is 55. The ascent profile of a mission determines if one or two OMS thrusting periods are used to put in orbit. Before the first OMS firing, the remaining LH₂ and LOX trapped in the main propulsion system ducts are dumped. In each OMS fuel line, there are two spring-loaded solenoid valves that close the lines. Pressurized nitrogen gas, from a small tank located near the engine, opens the valves and allows the fuel and oxidizer to flow into the combustion chamber of the engine. When the engines shut off, the nitrogen goes from the valves into the fuel lines momentarily to flush the lines of any remaining fuel and oxidizer; this purge of the line prevents any unwanted explosions. During a single flight, there is enough nitrogen to open the valves and purge the lines 10 times. Either one or both of the OMS engines can fire, depending upon the orbital manoeuvre needed. For vehicle velocity changes in the range 1.2 m/s normally only one OMS engine is used (the Shuttle needs some $\Delta v = 3.3$ m/s to raise the orbit 10 km). The OMS engines together can accelerate the shuttle by 0.6 m/s². To place the Shuttle into orbit takes about $\Delta v = 50..150$ m/s, and another $\Delta v = 150$ m/s to deorbit it (both OMS rockets can be used for up to 1250 s burn-time). OMS engines are designed to start and stop 1000 times in total life, and 15 h total burn-time. Cooling method is fuel regenerative for the combustion chamber, and radiative for the nozzle, which is made of a columbium alloy.

The RCS is used to provide attitude hold and minor translation manoeuvres as required for on-orbit operations and rendezvous with the ISS. The flight crew can select primary or vernier RCS thrusters for attitude control on orbit. Normally, the vernier RCS thrusters are selected for on-orbit attitude hold.

Reentry events
Orbiter reentry is initiated by firing the OMS, a time named TIG (time of ignition to ground), what initiates a no-return trajectory to the surface (well, the OMS might be used to regain speed and altitude for a while, if another 3000 kg of propellants were available, about 1500 kg to regain the LEO, and 1500 kg for a new deorbit firing). A coarse timeline for reentry events may be:

- TIG-3 h. Reentry preparation starts (about 4 hours before touchdown) when the shuttle is orbiting in a forward-looking upside-down attitude at an altitude of some 320 km, with an orbit inclination that depends on the mission (from $i = 28.5^\circ$ used for GEO satellites, the minimum from KSC, to $i = 52^\circ$ used for ISS rendezvous). Crewmembers begin to configure the on-board computers for reentry (the sequence can be executed automatically or manually), as well as the hydraulic system that powers the shuttle's wing-flaps and rudder.
- TIG-2 h. Payload bay doors are closed. Later on, Mission Control gives a go-ahead to proceed.
- TIG-1 h. Crewmembers put on their orange spacesuits and strap themselves into their seats. The Shuttle rotates 180° to point the OMS-nozzles forward (in the advancing direction), while flying upside down. The APU is started.
- TIG. Deorbit burn starts: the two OMS engines are used for about 3 minutes, to get a delta-v of 100..110 m/s. At this time, the Shuttle had been let to fall to about 280 km altitude, and its relative speed is 7.4 km/s, being almost at the antipodes of Florida. The Shuttle, which is advancing back-
side-first and upside-down in a slow diving (Fig. 8), flips over 180° (by pushing its nose down) to gain a forward upright attitude.

- TIG+0.5 h. The entry interface (EI) is crossed (NASA sets it at 400 kft, 122 km, but the difference with the 100 km Kármán-line crossing is not great (the rate of descent at these altitudes is almost 0.2 km/s).
- TIG+1 h. Touchdown. But this last half-an-hour, the atmospheric phase, merits a more detailed account, which follows.

The reentry ground track depends on the orbital plane of the mission (always coming from West to East): for $i=28.5^\circ$ the Shuttle enters the USA by the California border, but for $i=52^\circ$ it may enter by West Canada or by the South Pacific Ocean. The generic events are as follows:

- While crossing the entry interface at about $z=100$ km altitude, the Shuttle's aft steering jets are used to maintain an angle of attack of $\alpha=40^\circ$ nose-up attitude (causing a high air-drag), not only to slow it down to landing speed, but also to reduce reentry heating by getting the shock wave detached from the body. Below $z=85$ km, attitude control is by aerodynamic forces (the ailerons become effective at a dynamic pressure of $>500$ Pa), and then the Shuttle performs a series of four steep banks (each lasting several minutes), rolling over as much as 80º to one side and then to the other (maintaining $\alpha=40^\circ$); this swaying motion gives the shuttle's landing trajectory an elongated wavy ground track (S-shapes). There are two reasons for this banking descent:
  - In a straight line, its large angle of attack, $\alpha=40^\circ$ (nose-up attitude), would cause the descent angle to flatten-out, or even rise.
  - In this banking way, it dissipated energy sideways rather than just downwards.

- Blackout. While falling from 75 km to 60 km, the hot ionized gases of the atmosphere that surround the Orbiter (bright flashes can be seen through the crew-cabin windows) prevent radio communication with the ground for about 10 minutes (ionization blackout). Columbia's accident took place during this period, some 70 km above the California/Nevada border aiming at KSC, when a hole some 0.2 m in size in the thermal protection of the left-wing leading-edge, then at 2000 K and $M=23$, caused structural overheating, wing failure, and complete disaster. The blackout is not complete, however, and a radio link to a satellite can be established in the backward direction (there is no shock wave in the wake).

- At about 45 km altitude, and some 15 minutes and 700 km to the runway, flying at 3.5 km/s with angle of attack of about 35º, the Shuttle picks up a radio beacon from the runway (Tactical Air Navigation System) that provides precise range distance, entering the gliding-approach phase.

- At about 25 km altitude, 6 minutes and 100 km to the runway, its path angle changed from about $\gamma=1^\circ$ (almost level flight) to about 10º (a large descent rate, with noticeable nose down) while travelling at about 750 m/s ($M=2.5$).

- At about 15 km altitude, 5 minutes and 40 km from the runway, the flight becomes subsonic ($M<1$), and the commander takes manual control (the auto-pilot capability was never used), while the Shuttle drops at almost 100 m/s with the nose pointing almost 20º down. The sonic boom could be heard across Florida (as two claps a fraction of a second apart, one due to the nose shock and the other to the wing shock), but it was not audible when flying at greater altitudes.
• At about 3 km altitude, 2 minutes and 12 km from the runway, aerodynamic braking is applied to help further slowdown the vehicle, deciding to aim at one end or the other of the runway (there is no go-around choice), and beginning the approach-and-landing phase. To discourage birds flying nearby, some pyrotechnics and noise-making devices are used in the landing site.

• At about 600 m altitude, some 30 s before touchdown, the commander sharply raises the nose (the angle of attack) and slows the rate of descent (to 2..3 m/s) and the advance speed (from some 250 m/s to some 100 m/s at touchdown); the landing gears (main and nose) are deployed when the speed is around 130 m/s.

• At touch-down, back wheels first, a 12 m in diameter drag-chute (first used in 1991) is deployed to further reduce the speed (350 km/h at touchdown) and shorten the landing run. The front wheel touches down while at 80 m/s, and becomes the primary steering means. The chute is jettisoned once the orbiter slows to about 30 m/s, and the final rolling to full stop is by wheels braking.

• After landing stops, the crew goes through the shutdown procedures to power down the spacecraft, what takes about 20 minutes; during this time, the external surface cools down, and noxious gases, which were generated during the reentry heating, fade away. Finally, Shuttle control is handed from JSC to KSC, and it is towed out of the runway (by a diesel-powered tractor) to its landing facility some 3 km away.

The empty Orbiter has 78 000 kg, with maximum reentry payload of 14 400 kg (110 000 kg MTOM), able to carry 25 000 kg to LEO (only 13 000 to sun-synchronous polar orbit, or 3800 kg to GTO).

OTHER SPACECRAFT PROPULSION SYSTEMS

ATV propulsion

The Automated Transfer Vehicle (ATV) is an unmanned (but habitable) pressurised expendable cargo-spacecraft designed by ESA to resupply the International Space Station (ISS) with consumables, and to provide ISS propulsion. It is 10 m long with 4.5 m in diameter (48 m³ of pressurised volume), and 21 000 kg at launch (8000 kg payload).

The ISS (about 450 000 kg) requires an average of 6000 kg of propellant each year for altitude maintenance (it would fall some 30 km/yr by residual air-drag, depending on altitude, which must be in the range 300..460 km; it is typically within 330..430 km), and also for debris avoidance and attitude control. The ISS itself only has small thrusters, the major ones being the two main engines on Zvezda Service Module first fired in 2007 (although they were there since year 2000 when Zvezda made the ISS habitable); during the first years of the ISS, propulsion was provided by the Zarya module, now used mostly for storage (including propellants). Main propulsion for the ISS is provided by visiting expendable Progress spacecraft using UDMH/NTO propulsion (derived from Soyuz spacecraft, and launched with the Soyuz rocket), and alternatively by European ATV, Japanese Kounotori vehicles, or American Dragon and Cygnus spacecraft. Cygnus, from Orbital Science, carries a payload of 1260 kg to the ISS, and takes 1000 kg of ISS waste for disposal on a destructive reentry; it is launched by an Antares rocket ($m=240$ t, $L=50$ m, $D=4$ m, $m_{pay}=6$ t to LEO, with two NK-33 liquid rockets RP1/LOX for the first stage, and a solid rocket Castor-30 for the
second stage). All propulsive modules must attach at the rear of the ISS, to provide a rising push along the ISS velocity vector.

Typically, re-boosts are done once a month, firing thrusters with a total of about 1 kN for some 1000 s to impose a delta-v of just over 2 m/s (about $\Delta v=30$ m/s per year is needed to compensate the thin atmosphere drag on the ISS at 400 km altitude). As a check, using Tsiolkovski equation (3), with a $\Delta v=30$ m/s per year, $v_e=3350$ m/s for MMH/N$_2$O$_4$, and $m_{ISS}=450$ 000 kg, yields $m_p=450\cdot30/3350=4$ t per year of propellants. Propellant burn rate for $F=1$ kN is $\dot{m}_p = F/v_e = 1000/3350 = 0.3$ kg/s, so that the 4 t/yr take 13 000 s to burn, or about 1100 s if fired once a month.

The ATV propulsion system is contained in the unpressurised service module (Fig. 16), located aft of the habitable pressurised module. It comprises four main liquid-bipropellant engines of 490 N each for navigation, and 28 attitude control thrusters of 200 N each. All rockets use the same bipropellant, MMH/N$_2$O$_4$, stored in 8 titanium tanks with 7000 kg total capacity, pressurised with helium at 31 MPa (there are two carbon fibre-wound helium vessels). Main engines are R-4D-11 made by Aerojet, a rocket derivative of the Reaction Control System thrusters flown on the Apollo lunar and service module, and Shuttle's OMS. The 28 small thrusters (arranged in 4 clusters of 2, and 4 clusters of 5) are needed for the precision attitude control system (ACS) used in the autonomous rendezvous system.

An ATV derived service module (ATV-SM) is being developed for the Orion spacecraft. A shorter version of ATV (2.7 m long, 4.5 m diameter) will be placed between the launcher and the Orion crew command module, and remain attached to the Orion module until the capsule returns to Earth. Two interfacing adapters: to Orion, and to the launcher. The Orion main engine (26 kN) will go through the ATV-SM which will have 8 thrusters of its own (60 N each). Thermal control of ATV-SM will be by fluid loop instead of heat pipes as in ATV, with more radiators and MLI.
Meteosat and AlphaBus propulsion

Meteosat is a European series of geostationary meteorological satellites (since 1977). AlphaBus is a European platform for high-power (12..18 kW) geostationary communications satellites (first flight in 2013). Both Meteosat third generation (MTG) and AlphaBus use a unified propulsion system (UPS, Fig. 17d) with a 400..500 N apogee engine and sixteen small (10 N each) control thrusters. All are liquid bipropellant MMH/N₂O₄ rockets fed from two common propellant tanks. In AlphaBus the tanks are 1.6 m³ for MMH and 1.9 m³ for N₂O₄, each about 1.6 m in diameter (made of fibre overwrapped), operating in pressure regulated mode using two smaller helium tanks loaded at 30 MPa. In MTG the tanks are about half in capacity.

The first-generation Meteosat (MFG, 2.1 m diameter, 700 kg launch mass, 320 kg dry, 200 W, spin stabilised at 100 rpm) was equipped with two independent propulsion systems: a solid-propellant apogee boost motor, MAGE-1, and a small hydrazine propulsion system for orbit and attitude (spin and nutation) control. The second Meteosat generation (MSG, 2000 kg satellites) already combines the two propulsive tasks in one common tankage and feed system based on MMH/N₂O₄ rockets (two 400 N and six 10 N thrusters, with four propellant-tanks); a precursor of UPS but still operating at 100 rpm.

The second-generation Meteosat (MSG, 3.2 m diameter, 2000 kg launch mass, 900 kg dry, 600 W, spin stabilised at 100 rpm) is equipped with a unified liquid bipropellant propulsion system that feeds the apogee motor and all of the attitude and orbit control thrusters, with two 400 N apogee rockets used for a 3-burn GTO-to-GEO insertion, spending 800 kg of the 975 kg of propellant (stored in 4 tanks), and six 10 N thrusters. Two NiCd batteries provide 1200 Wh for ecliptic operations support.

The third-generation Meteosat (MTG), planned for 2018, will consist of two parallel positioned satellites, the MTG-I imager and the MTG-S sounder. Unlike the first and second generation Meteosat series, MTG will be based on three axes stabilised platform. The MTG-I will have 3.2 m diameter, 3000 kg launch mass, 700 kg dry, 600 W, spin stabilised at 100 rpm) is equipped with a unified liquid bipropellant propulsion system.

![Diagram of spacecraft propulsion](image-url)
Galileo system propulsion

Galileo is a constellation of navigation satellites. Each unit has one monopropellant tank with 75 kg of hydrazine (for 12 yr operations) and $8 \times 1$ N thrusters. The propulsion subsystem is designed for operation in blow-down mode. The beginning-of-life pressure is 2.5 MPa at 50 °C, whilst the end-of-life pressure is 0.55 MPa at 12 °C. Positive propellant expulsion is achieved by an EPDM-based, silica-free rubber diaphragm, retained at the mid-plane of the propellant tank sphere.

A trip to Mars (MLS-Curiosity)

As an example of all the propulsion needs involved in a complete interplanetary trip, we summarise here some facts related to the Mars Science Laboratory (MSL) mission, a NASA unmanned spacecraft which landed the Curiosity rover on Mars in 2012.

MLS was launched by an Atlas V two stage rocket (531 t total lift-off mass), with a booster powered by a single RD-180 engine, four solid rocket boosters, and one Centaur second stage, with a 5 m diameter payload fairing protecting the 3500 kg MSL. The interplanetary trip covered a distance of $566 \times 10^6$ km in 253 days, fine-tuned by eight thrusters using hydrazine fuel in two titanium tanks, with electrical power from solar panels on the rear base of a the slice-shape axially spinning craft, and radioisotope thermoelectric generators.

Notice that interplanetary trajectories are dominated by Sun gravity, and much more energy would be required to push 'uphill' away from the Sun, than letting the Sun to attract the spacecraft (in an elliptic orbit; avoiding a too-close flyby), and meeting the other planet at the aphelion of the orbit (previously computing that Mars will be there at the time of the spacecraft arrival). That is why an Earth-Mars trip is not going straight when both planets are the closest, at only $56 \times 10^6$ km (perihelion opposition), but goes along a path even longer than the farthest Earth-Mars distance ($400 \times 10^6$ km), as shown in Fig. 18.
Landing a rover on a planet with an atmosphere is difficult because the wheels must be protected from aerodynamic heating by a heat shield and later deployed. If a delicate landing, without too much dust blown by retrorockets, and already deployed wheels, is envisaged, the landing cannot be just by rockets like on the first Mars landers (Viking, 1976), neither by non-propulsive means (parachutes and airbags), like in MER’s Opportunity, in 2004. The sequence of events on entry, descent, and landing (a 540 kg cruise-service stage had separated 17 min, 5000 km, before EDL started), are:

- At about \( h=80 \) km altitude, one may consider that Mars entry starts; this is the Kármán line, with a corresponding circular-orbit speed of \( v = \sqrt{\frac{GM_{\text{Mars}}}{R_{\text{Mars}}}} = \sqrt{\frac{6.67 \times 10^{-11} \cdot 0.64 \times 10^{24}}{(3.4 \times 10^8)}} = 3.5 \) km/s. The spacecraft arrives from an Earth-Mars transfer orbit at 5.8 km/s, with a 4.5 m diameter heat-shield at the front (the largest ever flown in space). It enters the atmosphere with a relative flight angle of about 12° that parabolically approaches 0. It decelerates by atmospheric drag from almost the same 5.8 km/s at \( h=50 \) km, to about 600 m/s at \( h=10 \) km in 4 minutes, with peak deceleration of 12g at \( h=25 \) km (4.6 km/s). The hottest heat-shield temperature, 2400 K, occurs before, at \( h=30 \) km while at 5.5 km/s (inside the lander there are 10 ºC or so). This is a lifting descent with a trim angle achieved by having the centre of mass off-axis (two 75 kg tungsten peripheral masses were jettisoned minutes before atmospheric entry); the lift vector is controlled by four sets of two reaction control system (RCS) thrusters that produced approximately \( F=500 \) N per pair.

- At about \( h=10 \) km the parachute deploys, when travelling at about 600 m/s (\( M=1.8 \)), and soon after the heat shield pops off and drops away. The radar navigator takes command, and marks when to start powered descent (if it were done too early, it would run out of fuel; if too late, a crash). Prior to parachute deployment the entry vehicle ejects more ballast mass (six 25 kg tungsten weights) such that the centre-of-mass offset is removed. Below \( h=3.7 \) km, a camera beneath the rover acquires about 5 frames per second (with resolution of 1600×1200 pixels) during a period of about 2 minutes until the rover sensors confirmed successful landing.

- At about \( h=1.8 \) km, still travelling at about 100 m/s, 2 minutes after parachute opened, the rover-and-descent stage dropped out of the aeroshell-and-parachute module, first in a free fall lasting some 80 s, and later in a sideway powered flight to avoid the aeroshell-and-parachute falling on top of the rover, when landing. The descent stage is a platform above the rover with eight variable thrust (from 400 N to 3000 N) monopropellant hydrazine rocket thrusters on arms extending around this platform.
• At about $h=0.5$ km, and at 30 m/s, retrorocket thrust increases, and the rover deploys from its stowed flight configuration to a landing configuration while being lowered beneath the descent stage by the "sky crane" system (never used in space missions before). Four of the eight retrorockets are shut off before the bridle (three nylon ropes) and umbilical cord (with power and data connections) begin to spool out up to 7.5 m long, with Curiosity (899 kg, 80 of which scientific instruments) touching down at about 0.5 m/s with its six wheels (with shock absorbers).

• When tension on the cables reduces (during about 2 s), the rover knows it is on land, sends commands to the descent stage, and cuts the bridle and umbilical (with pyrotechnic devices) to let free itself, before the descent stage flights away some 150 m and crashes (total sky-crane propellant was 390 kg). Curiosity's computer switches from EDL mode to surface mode. All the auxiliary entry-and-descent equipment was 2400 kg. Although there were permanent on-line communication with Earth (through other Mars-orbiting relay satellites), it takes 14 minutes for signals to travel such a long way home.

On Mars' surface, Curiosity propels itself by electrical motors on each of its six wheels, with electricity generated by radioisotope decay (RTG) using 4.8 kg of plutonium-238 dioxide, designed to produce 125 W of electrical power from about 2000 W of thermal power at the start of the mission (100 W of electrical after 14 years).

ROCKET EMISSIONS AND OTHER ENVIRONMENTAL EFFECTS

We can split these effects by location: Earth bound, and Space bound. We focus here on the former, but the effects on the surface and atmosphere of visiting celestial bodies should not be neglected (possible biological contamination, chemical pollution by emissions and wastes, mechanical and radiative effects of debris and dust...). We further restrict the subject to ground effects caused by rockets used for space propulsion, leaving apart other rocket uses.

On ground, the environmental impact may range from the accidental fall of large spacecraft debris in populated areas, to the pollution caused by normal spacefaring, among which one may consider the effect of emissions from the almost one hundred orbital launches per year (and several dozen suborbital launches related to space research). These present launchers consume about $40 \cdot 10^6$ kg/year of propellants, most of it within the atmosphere (about a third at the troposphere), producing an equal amount of exhaust gases (with particulate matter), mainly $\text{CO}_2$ and $\text{H}_2\text{O}$, and to a lesser extent soot particles, $\text{Al}_2\text{O}_3$ particles, chloride compounds (HCl, AlCl₃), and nitrogen oxides (NOₓ). Globally, the amount of rocket emissions is very small, only about 0.01% of the fuel burned in aviation, which is just over 2% of our global $\text{CO}_2$ emissions. But all contributors to pollution must be aware of their share and be charged with a proportional responsibility (to pay for it and to search for improvements). Besides, spacefaring is the only human activity to directly put contaminants in the stratosphere (where they tend to accumulate because of their long lifetime in this stable layer; the residence time is 3–5 years).

Besides local chemical pollution near the launch site (always sparsely populated areas with wide restrictions), presently the largest global concern on launch emissions seems to be the effect of soot deposition in the stratosphere (emitted by RP1/LOX rockets), where it adds to the global warming because
its absorption of infrared radiation from the surface outweighs the cooling effect of increased solar reflection; notice that, by unit mass of propellant, launchers produce more than 1000 times more soot than modern turbofans (which emit around 0.03 g/kg). A second concern is the stratospheric ozone depletion by chloride compounds emitted by solid-propellant rockets (using NH₄ClO₄).

At the present stage of spacefaring, the noise and other local disturbances caused by a space launch are public attraction events rather the annoying aircraft noise, in spite of the strong roaring at lift-off that would kill people nearby if unprotected; it was about 190 dB on Space Shuttle launches, caused by the shock waves created by the supersonic exhaust jet when mixing with ambient air (the crew is only exposed to rocket noise for the first minute or so, until the vehicle becomes supersonic).

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