THERMAL MACHINES AND HEAT ENGINES

THERMAL MACHINES

A machine is a mechanical device with moving parts that helps to do some useful action, usually work, from a simple wheel-and-shaft, to a nuclear power station. Machines may be just mechanical (pulley-and-belt), hydraulic (water wheel, the earliest engine), pneumatic (windmill), electrical (electric motor, the most common nowadays), chemical (fuel cell), or thermal.

We restrict the name 'thermal machines' to devices that converts heat to work (heat engine), and to devices that pump low-temperature energy to high-temperature energy (by using some source of exergy), all working cyclically. The heat pumping machine may be intended to produce cold (refrigerator), or to produce heat (heat pump), or both at the same time (refrigerator with heat recovery, or heat pump). We do not include in this definition a simple thermal component as a compressor or turbine, in spite of the fact that it is a machine and it encompasses thermal processes.

Notice also that Engine Thermodynamics is a wider subject than Heat-Engine Thermodynamics; e.g. fuel cells are not heat engines but electrochemical engines, and internal combustion engines are really thermochemical engines, their study requiring Chemical Thermodynamics; in the strict sense, only steam engines (and Stirling engines and the like) are heat engines. We present in this chapter the thermodynamics of heat engines, and leave the thermodynamics of refrigerators and heat pumps for the next chapter. A broader perspective of power generation can be found under Thermal Systems. Moreover, thermoelectric generators, which are heat engines based on Seebeck effect, are not covered here.
THE HEAT ENGINE

What it is

A heat engine is a machine that produces work from heat, like the steam engine, the work-horse of the Industrial Revolution. Not all engines are heat engines (e.g. hydraulic wheels and windmills), but heat engines provide near 90% of the motive power generated in the world (an average of $2 \cdot 10^{12}$ W in the year 2000, nearly half-and-half for electricity and transportation), the other 10% provided by hydroelectric power stations. Nearly 60% of all the world energy consumption is devoted to run heat engines, the rest being devoted to industrial and domestic heating. Engines did not change too much from the Neolithic-watermills and windmills until the development of the first type of heat engine, the steam engine, in the 18th century. Since then, heat engines have produced a huge change in society, particularly regarding personal mobility and goods transportation.

The principle of a heat engine was established in Chapter 3: a heat engine is a device where a working fluid performs four basic processes: heat input, hot expansion, heat rejection, and cold compression. By means of these internal processes, the heat engine gets heat from a hot source, produces some net work, and rejects the rest of the energy balance as heat to a colder heat reservoir. It was precisely the understanding of this general principle (the need of at least two heat reservoirs) and the optimisation of it (the finding that the maximum efficiency only depends on the extreme temperatures), that gave birth to Thermodynamics in the 19th century. A key point to keep in mind is that, in spite of the heat-engine efficiency being maximum when heat flow to/from the working fluid is without temperature drop, power production requires finite temperature jumps (recall: a Carnot engine produces no power; see Dynamic efficiency of heat engines, in Chapter 3)

Practically all heat engines rely on the compressibility of the working fluid (i.e. a gas or vapour), thermoelectric devices being an exception, and on the fact that a hot expansion delivers more work than that needed for a cold compression (of a gas, vapour or liquid).

What it is for

Motive power (engines and motors) are used as stand-alone plants to deliver motion or electricity to other systems, or within vehicles to provide motive power (propulsion) and auxiliary energy. There were nearly a billion ($10^9$) vehicles worldwide in 2000, more than 80% of them passenger cars. More than $2 \cdot 10^9$ motor vehicles have been manufactured from 1900 to 2000, and a similar amount is expected to be delivered from 2000 to 2020.

The steam engine is now only found in the largest power stations (nuclear and coal driven), and today the most common heat engine is the internal combustion reciprocating engine (for cars, trucks, ships, small aeroplanes, and stationary engines), with a third type, the gas turbine engine gaining ground (for most aircraft, the faster types of ship, modern power stations and combined power-and-heat stations).

The largest thermal power plants are vapour turbines, typically limited to 1000 MW per unit in nuclear power stations because of heat transfer limitations from the reactor (fuel-fired power stations are limited...
to some 400 MW per unit because of combustion intensity limitations). Gas turbines may also reach some 300 MW per unit, and the largest reciprocating engines are marine diesel engines up to 100 MW (the largest petrol engine is just 0.3 MW). Notice that when referring to a heat engine, its power is always to be understood as its output power (i.e. its mechanical or electrical power; the thermal power input, commonly referred to the low heating value of the fuel used, being typically three times larger).

Thermal aspects of heat engines

We are going to analyse the internal workings of heat engines, but only from the thermodynamic point-of-view (i.e. what thermodynamic processes take place in actual heat engines), but not considering other important aspects of heat engines, either thermal or non-thermal. Thermal problems in power-generation equipment include heat transfer problems, particularly cooling problems, which are often crucial to all kinds of engines, not only in high-temperature turbine-blades for gas turbines, valves and walls cooling in reciprocating engines, but in more general heat dissipation problems due to friction, problems of sealing and interference due to thermal expansion (losses, wear, noise, cracks), etc. Besides, the study of heat engines is usually associated to that of fuels and their actual combustion process, the materials involved (including the required cooling), the mechanisms and their lubrication, the piping of the working fluid (air admission, fuel injection, and product exhaust), the electronic system (sensors and controls), testing and diagnostic means, and so on. And each of the mentioned aspects may demand ever-increasing efforts to analyse; e.g. in a few milliseconds, a thin liquid stream in turbulent motion may cavitate and flash-boil at the tip of the fuel injector, form a spray, break it down to a myriad of droplets, which evaporate, ignite, burn, and generate unwanted emissions. It is understood that knowledge of the basic principles of heat engines is not enough to design a working machine, not even to make it running (but would help a lot).

Although a more general presentation of heat engines is here provided, the proposed exercises are basically limited to ideal cycles with few practical modifications (e.g. isentropic efficiencies in turbo-machinery, temperature jumps in heat exchangers, and so on).

Carnot cycle

With his 1824 masterpiece "Réflexions sur la puissance motrice du feu, et sur les machines propres a développer cette puissance", Nicolas Leonard Sadi Carnot was the first to provide a thermodynamic model of a heat engine, abstracting from the only available heat engine, the steam engine, to pinpoint the fundamentals: the idea of a generic working fluid, performing a generic cyclic process, interacting with generic heat reservoirs. In that his only publication, Carnot concluded that all heat engines where limited in their energy-conversion efficiency by the operating temperatures, and that the maximum efficiency is obtained when the working fluid is assumed to follow four ideal processes (Fig. 17.1):

- An isentropic compression (1 to 2), to change temperature without heat transfer.
- An isothermal heat input (2 to 3), from the hot source, at the hot-source temperature.
- An isentropic expansion (3 to 4), to change temperature without heat transfer.
- An isothermal heat rejection to the cold source (usually the environment), at the cold-source temperature.
Carnot reached those conclusion by a set of rational deductions, namely: 'any engine with friction would have less efficiency than one without', 'among all engines exchanging heat at different temperatures, the one with highest efficiency only exchanges heat at the two extreme temperatures (the hottest and the coldest)', and 'all reversible engines working with the same couple of temperature extremes have the same efficiency'.

The energy and exergy efficiencies were defined in Chapter 3, and can be deduced by establishing the overall energy conservation, \( \Delta E_{\text{univ}} = Q_{\text{hot}} - W - Q_{\text{cold}} = 0 \), and the overall entropy balance, \( \Delta S_{\text{univ}} = -\frac{Q_{\text{hot}}}{T_{\text{hot}}} + \frac{Q_{\text{cold}}}{T_{\text{cold}}} \geq 0 \), the latter being zero in the ideal case of a Carnot cycle, what yields:

\[
\eta_e = \frac{W_{\text{net}}}{Q_{\text{pos}}} \quad \Rightarrow \quad \eta_{e,Carnot} = 1 - \frac{T_1}{T_2}, \quad \eta_s = \frac{\eta_e}{\eta_{e,Carnot}} \quad \Rightarrow \quad \eta_{s,Carnot} = 1
\]  

(17.1)

Carnot cycle, and any other conceivable power cycle, is to be run clockwise in both the \( T-s \) and \( p-V \) thermodynamic diagrams, since the heat engine receives net heat, that in a reversible process is \( Q_{\text{net}} = \int TdS \) (recall the area interpretation of integrals), and delivers net work, that in a reversible process is \( W_{\text{net}} = \int pdV \); notice that in a cycle \( \int dU = \int TdS - \int pdV = 0 \).

The Carnot cycle is not practical, not only because of unavoidable frictional losses (could be minimised with appropriate lubrication), but because of the heat transfer with negligible temperature jump, that would render the heat transfer rate infinitesimal for a finite size engine with finite thermal transmittance with the heat sources. However, heat transfer rates may be highly increased if the gas moves through a porous media that serves as intermediate heat source, as in the cycles of Stirling and Veuillemier.

**GAS POWER CYCLES**

Inside a heat engine, a working fluid cyclically evolves taking heat from a hot source and rejecting heat to the environment, producing some mechanical work in the way (usually shaft work). Gases are the best working substances because of their ease to exchange thermal to mechanical energy by compression or expansion, whereas liquids have little compressibility. We call ‘gas cycles’ those where the working substance stays all the time in the gas phase, and vapour cycles those where the gas condenses to liquid in some part of the cycle (and back again). Although any cycle may in principle be used as a heat engine or as a refrigerator and heat pump by just reversing the direction of the process, in practice there are big difference and the study is split between power cycles and refrigeration cycles.
Many gas cycles have been proposed, and several are currently used, to model real heat engines: the Otto cycle (approximates the actual petrol engine), the Diesel cycle (approximates the actual diesel engine), the mixed cycle (an hybrid of the last two that is better than both), the Brayton cycle (approximates very well the actual gas turbine engine), and the Stirling cycle (used in some exotic applications). We restrict the analysis to the so called ‘air standard’ model, which assumes air with temperature-independent properties as the working fluid, neglecting fuel addition effects (the air to fuel mass ratio is around 15:1 in Otto engines, 30:1 in Diesel engines, and 50:1 in Brayton engines), and consider an equivalent heat addition.

**Otto cycle**

The Otto cycle is a first approximation to model the operation of a spark-ignition engine, first built by Nikolaus Otto in 1876, and used in many cars, small planes and small power systems (below say 200 kW) down to miniature engines. This is a reciprocating internal combustion gas engine (the master in the 19th c. was the external combustion steam engine), typically using gasoline as fuel. The Otto engine is sketched in Fig. 17.2, where the typical terms are introduced for engine-geometry characteristics (stroke, bore, displacement and compression ratio); other terms for engine-operation characteristics are shaft speed, mean pressure, power, fuel consumption, torque, volumetric efficiency, energy efficiency, etc. The intake has traditionally being a homogeneous mixture of air and fuel, but direct fuel injection in the cylinders is taking over.

[Fig. 17.2. Sketch and nomenclature for reciprocating engines: a) 4-stroke, b) 2-stroke (uni-flow).]

In the ideal air-standard Otto cycle, the working fluid is just air, which is assumed to follow four processes (Fig. 17.3): isentropic compression, constant-volume heat input from the hot source, isentropic expansion, and constant-volume heat rejection to the environment.

[Fig. 17.3. The ideal Otto cycle in the T-s and p-V diagram, and a practical p-V trace of four-stroke and two-stroke engines.]
The main parameters of both ideal and real Otto cycles are:

- **Size**, measured by the displacement volume (the volume swept by the piston, \(V_1-V_2\)), usually less than 0.5 litres per cylinder, to avoid self-ignition.
- **Speed**, more precisely crankshaft speed, \(n\), with a typical operation range \(n=1000..7000\) rpm \((n=20..120\) Hz\). The maximum value may be in the range \(n_{\text{max}}=6000..8000\) rpm for four-stroke engines. Two-stroke motorcycle engines run quicker \((n_{\text{max}}=13\) 000 rpm\), the quickest \((n_{\text{max}}=20\) 000 rpm\) being the smallest engines used in aircraft modelling (two stroke, 1 cm\(^3\), 200 W, using methanol or ether fuel with some 5..15% oil for lubrication and anticorrosion).
- **Compression ratio**, \(r=V_1/V_2\), with a typical range of \(r=8..11\) (up to 14 in direct-injection spark-ignition engines), limited by the 'knock' or self-ignition problem.
- **Mean effective pressure**, \(p_{\text{me}}\), defined as the unit work divided by the displacement, with a typical range of 0.2..1.5 MPa (the full-load value may range from \(p_{\text{me}}=1.2\) MPa in two-stroke motorcycle engines, to \(p_{\text{me}}=1.7\) MPa in the largest turbocharged engines). Maximum pressure may have a typical range of 4..10 MPa. Performance maps of reciprocating engines are usually presented on a \(p_{\text{me}}-n\) diagram, i.e. mean effective pressure versus engine speed.

The cold-air-standard model takes as working fluid air with constant properties (those at the inlet, i.e. cold), what renders the analysis simple. The energy exchanges for the trapped control mass, \(m\), are
\[
W_{12}/m=c_v(T_2-T_1), \quad Q_{12}=0, \quad W_{23}=0, \quad Q_{23}/m=c_v(T_3-T_2), \quad W_{34}/m=c_v(T_4-T_3), \quad Q_{34}=0, \quad W_{41}=0, \quad Q_{41}/m=c_v(T_1-T_4),
\]
and the energy and exergy efficiencies are:
\[
\eta_{e,\text{Otto}} = \frac{W_{34}-W_{12}}{Q_{23}} = \frac{(T_3-T_4)-(T_2-T_1)}{T_3-T_2} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r^{r-1}} \quad \text{and} \quad \eta_s = \frac{W}{W_{\text{max}}} = 1 - \frac{1-T_1}{1-T_3}. \quad (17.2)
\]

Real spark-ignition engines do not recycle the working fluid; they are all internal combustion engines where ambient air is pumped in, and fuel (a volatile liquid like gasoline, or a gas) is added to prepare a homogeneous reactive mixture that at a certain point (cinematically or electronically controlled) is ignited by an electrical spark (a minute spark is enough to ignite the easily flammable fuel/air mixture). Fuel addition is nowadays electronically controlled by injection, either at the inlet-duct of each cylinder, or directly inside the cylinder, as in a typical diesel engine (but with a much lower injection pressure, less than 10 MPa); at first, say from 1900 to 1980, the common fuel-addition system was the carburettor, based on venturi suction from a small float-regulated fuel chamber. Gaseous fuels, like coal-gas, liquefied petroleum gas (LPG), and natural gas, are also used instead of gasoline, as well as other liquid fuels like methanol, ethanol and ethers, usually added to gasoline up to a 20%. With the advent of direct injection in the cylinder with electronic control, mixture stratification has become possible, highly decreasing emissions and detonation risk at part loads (it was already suggested by Otto itself, but unfeasible at the time); the idea is to start with a lean dosage and end with a stoichiometric mixing close to the spark plug at ignition time, controlling engine power via fuel injection like in diesels, instead of by choking the main air flow as in homogeneous-mixture traditional Otto engines.
One of the key advantages of reciprocating internal combustion engines, both Otto and Diesel type, is that cheap materials can be used in their construction (e.g. cast iron, against very expensive nickel alloys in gas turbines), because peak temperatures (up to 3000 K) are only realised within the burning gases and for short times, with an average temperature during the whole cycle of some 700 K, which would be the quasi-steady temperature level at the wall (because of its much larger thermal inertia), even without cooling; engine-wall cooling is applied to lower this ‘adiabatic’ temperature down to 600 K to prevent thermal decomposition of lubricating oil.

Reciprocating engines may be two-strokes or four-strokes in a cycle; the simple to model is the four-strokes engine: one stroke is used to fill the cylinder with the air-fuel mixture through the inlet valve (the inside pressure is a little below atmospheric); a second stroke is used to compress the mixture with valves closed and, at a precise point close to maximum compression, burn it with a spark; a third stroke is used to let the product gasses to expand and do work on the crank-shaft; and a fourth stroke is used to force the burnt gasses out through the exhaust valve, and start a new cycle (two crank-shaft turns have elapsed). In two-stroke engines there is an overlap between intake and exhaust, with the inlet stream used to sweep the burnt gasses (throwing unburnt gases to the tailpipe!), and the whole cycle is performed in one turn of the crankshaft. In spite of the higher fuel consumption and pollutant emissions, and the difficulties for their lubrication (oil is added to the fuel), two-stroke engines have very high specific power, and thus they are used for small engines (in model vehicles, garden engines, motorcycles, and outboard boats). Valve design and operation is crucial in all reciprocating engines, but particularly so in two-stroke types. Spark are generated by high-voltage dielectric breakdown in the spark-plug gap, nowadays with local coils excited by solid-state electronics (e.g. thyristor trigger and Hall-effect distributor), and before (say from 1910 to 1980) with a central induction-coil, a mechanical contact breaker, and a mechanical distributor.

The upper limit in power and engine size, for homogeneous mixture engines, is dictated by the burning time, i.e. the time for propagation of the initial flame ball created around the spark, to the chamber corners. Actual propagation speeds are 20..30 m/s, proportional to the mean turbulence level (causing flame stretching); there is a fortunate correlation between this flame speed and engine speed, since turbulence is generated in the intake process, and increases with flow rate, so that the combustion process extends for nearly the same crank angle, around 30°, for all engine regimes. This turbulent flame speed is well above the laminar flame speed, which is below 0.5 m/s for all typical fuel mixtures (notice that just a 5 cm run in a car engine cylinder, would last 0.1 s at 0.5 m/s, but at 3000 rpm its crank shaft has gone 50·0.1=5 turns!).

Typical energy efficiencies of these engines are low, 25% to 35% when running at nominal power, and much lower at partial load (the main air flow is strangulated), but the engine is light, very powerful and responsive (accelerates very quickly), and less expensive than diesels. Pollutant emission may be high because premixed fuel cannot burn close to the walls (the effect of walls is smaller in larger diesel engines, and most of the nearby gas is just non-premixed air), making the exhaust catalyser an environmental need.

Example 1. Comparison among different power cycles

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Example 2. An Otto cycle engine

Diesel cycle

The Diesel cycle is a first approximation to model the operation of a compression-ignition engine, first
built by Rudolf Diesel in 1893, and used in nearly all boats (the first in 1903), nearly all trucks (the first in
1923), many locomotives (the first one in the 1940s, but taken over by electric drive after a few decades
of prominence), most cars nowadays (the first one in 1936, but it took decades to gain market), many
medium-large electric auxiliary-power and cogeneration systems, and even some small airplanes. It is the
reference engine from 50 kW to 50 MW, due to the fuel used (cheaper and safer than gasoline) and the
higher efficiency. This is a reciprocating internal combustion engine with the same sketch as in Fig. 17.2,
but substituting the spark-plug by the fuel injector (the fuel is injected at very high pressures, up to 200
MPa, to ensure immediate vaporisation). One of its key advantages compared to Otto engines is the great
load increase per cylinder associated to the higher pressures allowed (the mixture of fuel and air would
detonate in Otto engines at high compressions), and the further load increase associated to charging
previously-compressed air (turbocharging), nowadays implemented in practically all diesel engines.
Another advantage is the much better performance at part loads, since it is achieved by injecting less fuel
instead of by throttling, and the torque (power divided per angular speed, $M=P/\omega$), changes less with
angular speed.

In the ideal air-standard Diesel cycle, the working fluid is just air, which is assumed to follow four
processes (Fig. 17.4): isentropic compression, constant-pressure heat input from the hot source, isentropic
expansion, and constant-volume heat rejection to the environment.

Similarly to the Otto cycle, the main parameters of ideal and real Diesel cycles are also the size, measured
by the displacement volume (that may reach more than 1 m$^3$ per cylinder in large marine engines), the
compression ratio, $r=V_1/V_2$ (with a typical range of 16..22, limited just by strength), the cut-off ratio, $r_c$, or
the mean effective pressure (in the range 1..2 MPa), or the maximum pressure (in the range of 3 MPa to
20 MPa), and the speed (with a typical range of 100..6000 rpm). The energy efficiency can be expressed
as:

$$\eta_{\text{r,Diesel}} = 1 - \frac{1}{r_c^{\gamma-1}} \frac{\gamma - 1}{\gamma (r_c - 1)}$$

(17.3)

Real compression-ignition engines take ambient air (often after a first stage compression) and compress it
(inside the cylinder) so much, rising the temperature accordingly, that the fuel burns as it is injected (after
a small initial delay due to vapourisation and combustion kinetics). The external air compression is
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performed in a centrifugal compressor driven by a centrifugal turbine moved by the exhaust gasses (turbocharger); the air is cooled after external compression (inter-cooler) before further compression within the cylinders, to increase the efficiency, as explained in Multistage compression.

The higher pressures in Diesel than in Otto engines require a robust engine-frame and delicate fluid injection hydraulics (with injection pressure up to 200 MPa), but the wider range of fuels (from gas-oils to fuel-oils), their better safety (those fuels are combustible but not inflammable at ambient temperature), the better fuel control by direct injection, longer durability and better economy, are making Diesel engines to take over traditional gasoline-engines markets (for heavy-duty applications it has always been unrivalled). The fuel-injection system is the heart of a diesel engine, having to supply very precise minute amounts of liquid at a very high pressure through injection holes of some 0.1 mm in diameter and 1 mm long, where the liquid usually cavitates (it enhances spraying and burning); a great advance took place in late 20th century with development of the common-rail injection system, in which a very-high pressure reservoir (the common rail) separates the task of pumping from the task of metering (before that, the injection pump had to control pressure and fuel volume at the same time); the electronic metering has brought not only better fuel control, but a dependence on electricity for the working of diesel engines, as has always been for spark engines. Typical energy efficiencies are from 30% to 54% (based on the lower heating value of the fuel), the latter efficiency (the largest of any single thermal engine), being achieved in large two-stroke marine low-speed engines with bores larger than 0.5 m: first, because the thermal losses decrease with size, and second, because the very low speed (some 100 rpm instead of the typical 3000 rpm for a car engine) allows for a more complete combustion (more time to burn, and burning nearly without volume change) and decrease friction losses (in spite of the fact that the mean piston speed stays at some 6..7 m/s for the whole range of reciprocating engines: from the 1 cm³ 50 W model, to a 1 m³ 5 MW per cylinder 'three-store castle' large marine engine).

Most large Diesel engines are supercharged, i.e. fed with compressed air instead of atmospheric air, usually by means of a turbocharger (a small compressor shaft-coupled to a small turbine driven at very high speeds (up to 100 000 rpm) by the exhaust gasses; some 10% of the fuel energy goes through that shaft), with an intermediate cooling of the compressed air before intake to the cylinders (intercooler). The two-stroke cycle is better suited to Diesel engines, since only air is used to sweep the burnt gases (scavenging), and not fresh mixture, but, because of the difficulty in lubricating, it is only used in the largest marine engines (10 MW..100 MW), where residual fuel (must be preheated to flow) can be used.

Mixed cycle

The mixed (or dual, or Sabathé, or Seiliger, or 5-point) cycle, sketched in Fig. 17.5, is a refinement to both Otto and Diesel cycles, at the expense of an additional parameter, the heat-addition pressure ratio, $r_p = p_3/p_2$.

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Fig. 17.5. The dual or Sabathé cycle.
The energy efficiency can be expressed as:

$$\eta_{e,\text{dual}} = 1 - \frac{1}{\gamma} \left( \frac{r_p r_e^\gamma - 1}{\gamma r_p (r_e - 1) + r_p - 1} \right)$$  \hspace{1cm} (17.4)

**Brayton cycle**

The Brayton cycle, named after the American engineer George Brayton (that built a two-stroke reciprocating engine in 1876 and advanced combustion chambers at constant pressure), is a good model for the operation of a gas-turbine engine (first successfully tested by F. Whittle in 1937, and first applied by the Heinkel Aircraft Company in 1939), nowadays used by practically all aircraft except the smallest ones, by many fast boats, and increasingly been used for stationary power generation, particularly when both power and heat are of interest.

In the ideal air-standard Brayton cycle, the working fluid is just air, which is assumed to follow four processes (Fig. 17.6): isentropic compression, constant-pressure heat input from the hot source, isentropic expansion, and constant-pressure heat rejection to the environment. Contrary to reciprocating engines, the gas turbine is a rotatory device working at a nominal steady state (it can hardly work at partial loads); spark-plug ignition is used to start up combustion, since air compressor outlet temperature is not high enough to inflame the fuel injected (the correct air-flow being already provided by the compressor, either driven by an auxiliary power unit, or, in modern power plants, by reverse feeding the alternator to work as an electric motor).

![Fig. 17.6. The ideal Brayton cycle in the T-s and p-V diagram, and the regenerative Brayton cycle.](image)

Most gas turbines are internal combustion engines where the working fluid must be renovated continuously as sketched in Fig. 17.7, but some gas turbines use a closed-loop working fluid.

![Fig. 17.7. Open cycle and closed cycle gas turbine models.](image)

The main parameters of ideal and real Brayton cycles are the turbine-inlet temperature, $T_3$, the compressor pressure ratio, $\pi = p_2/p_1$ (within the range 4..40), the compressor and turbine efficiencies, and the size, measured by the air mass flow rate. The energy and exergy efficiencies for the ideal Brayton cycle (compressor and turbine efficiencies of 100%), can be expressed as:

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\[
\eta_{e,\text{Brayton}} = \frac{W_{\text{net}}}{Q_{\text{pos}}} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{\pi^{\frac{\gamma - 1}{\gamma}}} \quad \text{and} \quad \eta_{x,\text{Brayton}} = \frac{W}{W_{\text{max}}} = 1 - \frac{1 - \frac{T_1}{T_2}}{1 - \frac{T_1}{T_3}}
\] (17.5)

Real engine efficiencies are comparatively low, from 25% to 40% (it was only 15% when they started, in the 1950s), but in combination with a bottoming vapour cycle, they reach 50..59% based on LHV; combined cycle power plants are the present standard in electricity generation. Contrary to reciprocating and steam engines, the gas turbine can only work with fine-tuned components, since it gives no net power if the compressor and turbine efficiencies fall below say 80% (modern gas turbines can have compressor efficiencies of 80% to 88%, and turbine efficiencies of 88% to 90%).

The two factors that most affect gas-turbine efficiency are turbine-inlet temperature and pressure ratio, which should be the highest the possible. Turbine-inlet temperature has increased almost linearly from 1100 K in the 1950s to 1800 K in modern aircraft gas turbines with blade cooling. Pressure ratio has also increased almost linearly from 10:1 in the 1950s to 40:1 in modern aircraft gas turbines. It is easy to prove that for fixed extreme temperatures (ambient and turbine-inlet) there is a pressure ratio that maximises the work per unit mass flow rate, thus rendering the smallest engine for a given power, this optimum value being:

\[
\pi_{w_{\text{max}}} = \left( \frac{\eta_c \eta_T}{\pi^{\frac{\gamma}{\gamma - 1}}} \right) \left( \frac{T_2}{T_1} \right) \left( \frac{T_2}{T_1} \right)^{\frac{\gamma}{2(\gamma - 1)}}
\] (17.6)

where \( \eta_c \) and \( \eta_T \) are the compressor and turbine isentropic efficiencies, defined in Chapter 5 (5.28-29).

Several improvements to the simple Brayton cycle are in use. Besides the multistage compression and expansion explained in Chapter 5, the main variant is the regenerative cycle (Fig. 17.6), where heat from the exhaust gasses is used (from point 4 to 5) to heat up air before entering the combustion chamber (from point 2 to ideally up to point 2' in Fig. 17.6, although in practice the heat exchanger efficiency will limit this value). The heat recovery from the exhaust gasses may be also performed externally to the cycle, e.g. generating vapour in a heat exchanger (boiler), that may be directly used for heating applications or may even get expanded in a vapour turbine to produce further work (combined Brayton and Rankine cycles).

**Example 3. A Brayton cycle engine**

**Other gas cycles**

Practical gas engines are basically internal-combustion engines, either piston engines, or gas turbines. Their thermodynamic processes are best represented by the dual cycle and the Brayton cycle, respectively. But there are other gas cycles of interest in power generation, like the Stirling cycle, which is actually the first gas-cycle ever proposed, invented in 1816 by the Scottish clergyman Robert Stirling, the Atkinson cycle (patented in 1877 by Atkinson as a modification of the Otto cycle), and so on. The
Stirling engine is an external heat-input engine (can be driven by external combustion, solar radiation, and so on, like in the steam engine); it faded away in the 19th century against the steam engine (which was more practical), and later against the gas engines (that are more powerful), but it has got considerable attention lately as a non-polluting engine (if driven by solar energy or waste heat). Recall that combustion is just a mean to get high temperatures; any thermodynamic cycle (Otto, Diesel, Brayton...) can be made to run on solar energy, waste heat, etc. However, all gas power cycles require a large temperature difference in practice (e.g. concentrated solar power), and the only cycles to run with temperature differences of less than 100 K are the vapour cycles (see ORC, below).

The working substance in a Stirling engine may be just air, or better helium or hydrogen (to have the best thermal conductivity), and the four processes ideally followed are (see Fig. 17.8): from 1 to 2, isothermal compression with heat rejection to the environment; from 2 to 3, constant-volume compression by heat input from a heat regenerator (a porous media with a steady longitudinal temperature-ramp; the fluid enters cold at 2, and exits hot at 3); from 3 to 4, isothermal expansion with heat input from the hot source; for 4 to 1, constant-volume heat rejection to the regenerator (the gas now flows contrary to the 2-3 process). The regenerator divides the working gas in two regions: a hot one and a cold one, with a linear temperature variation between both ends (the porous matrix must be thermally insulating, to keep this gradient, but with a large contact area to enhance heat transfer with the gas). A Stirling engine may run without regenerator, but then the efficiency is very low. An auxiliary piston, the displacer, forces the gas at constant total-volume to move through the regenerator; the displacer has to overcome the pressure loss of the gas through the regenerator, but does not pressurises the gas, whose pressure rise is due to heating and not to pushing. The displacer is cinematically driven by the main crank-shaft mechanism, or dynamically driven by resonance.

![Fig. 17.8 The ideal Stirling, Ericson, Atkinson, and Lenoir cycles.](image)

Similar to the Stirling cycle is the Ericson cycle, where heat regeneration is isobaric instead of isometric as in the Stirling one (the regenerative Brayton cycle with infinitesimal multistage compressions and expansions approaches to the Ericson cycle). Notice that there is nothing essential to the four-process engines, and a three-process cycle (see Fig. 17.8), named after J.J.E. Lenoir, a pioneer of two-stroke engines in the 19th century, is used to model the operation of a pulse-reactor.
Energy and exergy efficiencies of 36% and 50% respectively have been reached with prototype Stirling engines of up to 10 kW, the main problems being the regenerator loss of efficiency at high speeds (>30 Hz, i.e. >1800 rpm), the radiant heat loses at high temperature (>1000 K), incomplete exchange of gas between the hot and cold zones, and leaking at high pressure (>5 MPa).

An old cycle that has gained interest lately is the Atkinson cycle (Fig. 17.8). The original Atkinson cycle was proposed in 1887 by James Atkinson as a modification of the Otto cycle with a prolonged power stroke, but had a complicated mechanism to provide a shorter admission/compression run and a longer expansion/exhaust run. The modern Atkinson cycle, used in most of the hybrid electric cars since 1997, is just a 4-stroke Otto cycle with highly-retarded closure of the intake valve; i.e. in its ideal $p-V$ sketch in Fig. 17.8, in the admission stroke the piston goes from 0 to 4; in the second stroke, there is some partial air evacuation at constant pressure (from 4 to 1) followed by compression from 1 to 2; in the third stroke, there is the heat release; and the fourth stroke is the expansion until ambient pressure (3 to 4). Note that there is some fresh-air charge rejected (from 4 to 1), lowering the compression ratio to about $r=8$, but leaving a larger expansion ratio, about $r=12$; instead of pushing this fresh-air back into the intake manifold, it can be directed to the exhaust manifold (opening the exhaust valve instead of leaving open the intake valve), with the advantage of some cooling on valves and walls; the engine must be of direct-injection type. Engines following the Atkinson cycle yield better fuel efficiency than Otto engines, but have smaller specific engine power, and consequently lower torque (but this is not a problem with its companion electric motor in hybrid cars).

A Miller cycle (patented in 1957) is an Atkinson cycle that uses supercharging, usually with a displacement compressor because it operates at low speed, either in spark-ignition or compression-ignition mode, and 2 or 4 stroke mode. Its efficiency is low at partial load.

**VAPOUR POWER CYCLES**

**Rankine cycle**

Most large electricity generating plants (central power stations), and very large ship engines, use water vapour (steam) as working fluid, following some variation of the basic Rankine cycle (named after the Scottish inventor William Rankine, that in 1859 wrote the first book on Thermodynamics), the only vapour power cycle in practical use since 1840 until in 1984 Alexander Kalina patented in the USA the cycle named after him.

The heat source for the boiler is usually the combustion products of a fuel (mainly coal) and air, or the primary refrigerant of a nuclear reactor, and the heat sink in the condenser is usually a water loop, open like in a river, or closed like in a cooling tower (as explained in Chapter 8). Thomas Newcomen is credited with the invention of the steam engine in 1705 for the purpose of driving the pumps used in clearing groundwater from mine shafts. Although the work-producing element was initially reciprocating cylinder-piston devices, in 1882 Gustav de Laval introduced the vapour turbine that has taken over.
Thermal machines and heat engines

Fig. 17.9. Carnot cycle within the two-phase region, basic Rankine cycle in the T-s and p-V diagram, and sketch of a vapour plant.

The four processes in a simple Rankine cycle are: isentropic compression of the liquid from 1 to 2 (Fig. 17.9), isobaric heating of the liquid, boiling and super-heating the vapour (from 2 to 3), isentropic expansion from 3 to 4, and isobaric heat rejection until full condensation of the vapour. The Rankine cycle is less efficient than the Carnot cycle (Fig. 17.9), but it is more practical since the compression is not in the two-phase region (see Chapter 6) and only requires a small work, and the expansion is mainly in the gaseous phase (high-speed droplets erode turbine blades). Water is not the ideal working substance because it changes phase at relatively low temperatures (below the critical point at 647 K), generating a lot of entropy in the heat transfer from typical high-temperatures heat-sources: 1000 K in nuclear reactors up to 2000 K in conventional combustion plants. Nevertheless, water is practically the only working substance used, because of its good thermal properties and availability. A caution note is that the right-hand-side end of the two-phase region, the saturated vapour line, that for water in the T-s diagram has the shape shown in Fig. 17.9, may be more vertical and even have negative slope at low temperatures for heavier molecular substances, naturally avoiding the problem of wet-vapour at the turbine.

Since, at ambient temperature (the heat sink), water change phase at lower-than-atmospheric pressure (e.g. 5 kPa at 33 ºC) the condenser must operate under vacuum and a so-called 'deaerator' is needed to remove non-condensable gasses from the feed water or infiltration; moreover, removal of oxygen and carbon dioxide in feed water is always desirable to avoid corrosions in the circuit. Gas solubility in a liquid decreases near the pure-liquid vapour saturation curve, thus deaeration may be achieved by heating the liquid at constant pressure (e.g. adding some vapour) or by making vacuum at constant temperature (e.g. with a small jet of vapour by venturi suction).

Maximum temperature in a steam power plant is limited by metallurgical constraints to less than 900 K (some 600 ºC), and the maximum pressure depends on the variations to the simple Rankine cycle used, with typical values of 10 MPa (supercritical Rankine cycles surpass 22 MPa). For a simple Rankine cycle with turbine exhaust above the vapour saturation line, the energy efficiency, with the perfect substance model, is:

$$\eta_{\text{Rankine}} = \frac{W_{\text{net}}}{Q_{\text{loss}}} = \frac{W_{3} - W_{0}}{Q_{\text{caldera}}} \approx \frac{h_{3} - h_{2}}{h_{3} - h_{2}} \text{no sat} \approx \frac{c_{p}(T_{3} - T_{4})}{h_{lv} + c_{p}(T_{3} - T_{2})}$$

where the work for pumping the liquid is neglected, $c_{p}$=2 kJ/(kg·K) is an average isobaric thermal capacity for steam, and $h_{lv}=$2400 kJ/kg the enthalpy of phase change at $T_{1}$ (a few degrees above the maximum cooling temperature); in most cases, however, the turbine exhaust is in the two-phase region, as
sketched in Fig. 17.9, state 4, with steam mass-fractions down to $x=0.9$, and $h_4$ in (17.7) must be found using the lever rule for liquid/vapour mixtures).

The main variants of the simple Rankine cycle are reheating (a multistage expansion as explained in Chapter 5) and regeneration (bleeding some vapour from the middle of the turbine, and before reheating if used, to heat the feed water). These feed-water heaters may be of the open or closed type (Fig. 17.10). In an open feed-water heater, steam extracted at some turbine stage is added to the main feed water stream (that must be previously pressurised to avoid boiling). In a closed feed-water heater, the extracted steam goes through the shell of a shell-and-tubes heat-exchanger and discharges in a lower-pressure heater or the condenser. The mass fraction of vapour to be extracted is designed to be able to heat the main feed-water stream until the saturation temperature of the extracted steam. Besides those water heaters, fuel-fired steam plants always incorporate a heat-recovery exchanger to preheat water from the condenser with flue gases, known as economiser. The ‘boiler’ itself (or steam generator) is usually a set of vertical tubes surrounding the hearth and connecting a lower liquid drum and a higher vapour drum, from which the steam goes through a superheater before entering the turbine.

![Fig. 17.10. Open and closed feed-water heaters.](image)

Steam turbines (notice that the term is used indistinguishable for the roto-dynamic device and for the whole power plant) are the largest thermal power plants, typically limited to 1000 MW per unit in nuclear power stations, with typical efficiencies from 30% to 40%, although supercritical power plants reach 45% (based on LHV). An advantage of steam turbines, extensive to all external combustion engines, is that any kind of fuel or other heat source may be used, contrary to internal combustion engines, where only fluid fuels, either residual to petroleum distillation but most of the times distillate fluids, can be used. The isentropic efficiency of the turbine is typically 85%, and the electromechanical efficiency of the alternator 98%. The energy and exergy balances in a typical steam power plant are presented in Table 1.

<table>
<thead>
<tr>
<th>Component</th>
<th>Energy output</th>
<th>Exergy use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combustion chamber</td>
<td>0</td>
<td>0.30</td>
</tr>
<tr>
<td>Boiler tubes heat transfer</td>
<td>0</td>
<td>0.30</td>
</tr>
<tr>
<td>Exhaust gasses (chimney)</td>
<td>0.15</td>
<td>0.01</td>
</tr>
<tr>
<td>Turbine</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>Condenser</td>
<td>0</td>
<td>0.03</td>
</tr>
<tr>
<td>Water cooling (condenser)</td>
<td>0.55</td>
<td>0.01</td>
</tr>
<tr>
<td>Shaft</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Example 4. Rankine cycle. Steam engine
Organic Rankine Cycles (ORC)

All large vapour power plants use water as working fluid, but water is not suitable for small steam engines with a low-temperature heat source (say $T_{\text{high}}<450$ K), because of its low vapour pressure at those temperatures. The only practical heat engines in this range make use of more volatile organic substances, what is known as organic Rankine cycle (ORC). Typical working substances are hydrocarbons, or halocarbons. There are commercial ORC engines powered by waste heat or solar heat (usually hot water at $>85$ ºC) generating some 50 kW in a 20 ºC environment, with an energy efficiency about $\eta=10\%$. The key problem in ORC design is the expander efficiency (friction loses on small piston or turbine devices).

As for the steam Rankine cycle, liquid vaporisation (the upper part in the cycle diagram) may go over the critical point of the working fluid, i.e. it can be transcritical (sometimes said supercritical).

Portable ORC engines driven by solar energy are competing with photovoltaic electric generators (much more expensive but simpler), and with internal combustion engines (much more polluting and noisy), in distributed and remote electricity generation.

Lorenz cycle and Kalina cycle

Most heat input/output to/from a plant's working fluid is from variable temperature heat sources/sinks, as the hot combustion gasses and the cold cooling water streams in the normal Rankine cycle. If, instead of using a pure fluid, a mixture were used in a Rankine cycle, due to its variable boiling/condensing temperature, the phase-change heating/cooling could better match the temperature rise/fall in the heating/cooling streams.

Similar to the Carnot cycle that optimises heat engines operating between two constant-temperature sources, the Lorenz cycle optimises heat engines operating between two gliding-temperature sources by adjusting the thermal capacity of the working fluid to that of the finite-capacity sources; i.e., Lorenz's cycle has four processes (like Carnot's cycle): isentropic compression, heating at constant thermal capacity matching that of the heat source (and its temperature variation), isentropic expansion, and cooling at constant thermal capacity matching that of the heat sink (and its temperature variation). Similar inverse Lorenz cycles apply for refrigeration and heat pumping.

However, the only vapour-mixture cycle developed has been the Kalina cycle, proposed by Alexander Kalina in 1984 (the first Kalina power plant, of 3 MW, opened in 1991, with several other similar or smaller plants built). Its characteristics are:

- An ammonia-water mixture (70%-NH$_3$ and 30%-H$_2$O) is used (a large know-how from absorption-refrigeration cycles existed).
- The turbine exit goes through a distillation and the heavy fraction (40%-NH$_3$ and 60%-H$_2$O) is condensed, and, once pressurised, the two fractions mix before entering the boiler again.
- Because temperature jumps across heat exchangers can be more uniform, the efficiency increases some 10% for a normal power station, but for special low-temperature applications more than 30%.
• Because ammonia lowers the boiling point, the Kalina cycle is better suited to low-temperature applications than the Rankine cycle, as in bottoming cycles (see below), geothermal power plants, and so on.

**COMBINED POWER CYCLES**

In a Rankine cycle, one single substance, like water, cannot easily match the high-temperature side (e.g. at the temperature of the combustion gasses, 1500 K to 2000 K, it is very difficult to transfer heat to water vapour), and the low-temperature side (to condense water vapour at ambient temperature is difficult because of the very low pressures and densities). The use of two Rankine cycles with different substances has been tried without success (an experimental plant was built with mercury for the top cycle and water for the bottom cycle).

The combination that has reached considerable success is the Brayton-Rankine combined cycle, where the exhaust gasses from a gas turbine are used to supply the heat in the boiler of a vapour turbine operating at not too-high temperatures. The Brayton-Kalina combination may be particularly successful in this respect. Natural-gas-fuelled combined power stations are the rule nowadays because of their low installation cost (some 450 $/kW against 1100 $/kW for coal stations), short-time operations start-up (2 years vs. 3.5 years for coal), and lower environmental impact (nuclear, coal and hydroelectric stations are on hold in Europe and USA), although wider fluctuation in gas price make the choice risky.

The turbo-diesel engine can also be considered a combined Diesel-Brayton cycle (Fig.17.11).

There are other combined cycles, still in the developing stage, that show great promise from both the energetics and the environmental aspects, like Graz cycle in Fig. 17.12 (proposed in 1985 by Prof. H. Jericha from Graz University, Austria), where pure oxygen is used as oxidiser instead of air (to avoid NOx-emissions and to have a pure exhaust gas), which on combustion with a fuel (e.g. natural gas) yields only water vapour and carbon dioxide as products, easily separated in a water condenser, ready to CO₂ capture, a major concern in the fight against global warming by anthropogenic greenhouse gases. Of course, CO₂ capture can be done with all traditional power cycles, e.g. by selective chemical absorption.
Thermal machines and heat engines

from the exhaust, but it is not competitive with oxy-fuel cycles like Graz's. Application of these air-independent propulsion concepts to submarines is evident.

Fig. 17.12. The Graz cycle, a new combined gas-steam power cycle burning fuel with pure oxygen, and capturing carbon dioxide.

Example 5. Combined Brayton-Rankine cycle

Besides the above-mentioned combined-power heat engines, a great promise lays in other combined-power plants, notably the coupling of gas turbines with fuel cells (and fuel reformers, both to make good use of coal as primary fuel through gasifiers, and to deliver coal-derived synthetic liquid-fuels).

PROPULSION

To propel or impel is to force a body to move forwards, and it requires a push (a force) and an energy expenditure (motive power) to overcome the drag imposed in practice by the nearby objects (if there were no interactions with the surroundings, a body could move without propulsion, as in outer space). Most propulsive systems aim at moving a body over land or through a fluid at constant speed, but all propulsive systems need to accelerate the body at some stage. The power required to propel a body of mass $m$ (e.g. a car) may be expressed as:

$$W_{\text{total}} = W_{\text{accel}} + W_{\text{climb}} + W_{\text{roll}} + W_{\text{fluid-drag}}$$

(17.8)

with:

$$W_{\text{accel}} = ma v, \quad W_{\text{climb}} = mg v \cos \theta, \quad W_{\text{roll}} = c_R mg v \cos \theta, \quad W_{\text{fluid-drag}} = c_D A_F \frac{1}{2} \rho v^3$$

(17.9)

where $a$ is the acceleration applied, $v$ the actual speed, $\theta$ the slope of climbing, $c_R$ a tyre-rolling coefficient (typically $10^{-2}$), $c_D$ a fluid-drag coefficient (typically of order 1, but may drop to 0.1 for streamlined bodies), $A_F$ the frontal area projected by the body in the direction of motion, and $\rho$ the density of the fluid medium.
The idealisation of a long rope being hauled in absence of any other interaction may be a good propulsion paradigm; because of force reciprocity, propulsion may be thought both as a pushing the environment (the rope, in the example) to the rear, and as pulling (the rope) from the front.

As a first model, for a constant-mass vehicle, the momentum balance in the direction of motion may be written as $m\ddot{x} = T - D$, $m$ being the mass of the body, $\ddot{x}$ its acceleration, $T$ the thrust force (pull or push on the rope example) and $D$ the drag force resisting the motion (friction with the rope, frictional and pressure drag in a fluid, etc.). For these vehicles, the drag can be measured by towing at constant speed, and the thrust by breaking. For steady motion at speed $v$, the energy dissipated just by the motion is $Dv = T\dot{v}$, and a propulsion efficiency may be defined to compare this energy with the energy supplied by the engine that cause the motion, $\dot{W}_{\text{shaft}}$ (because it is usually through a shaft), in the way:

$$\eta_p = \frac{T\dot{v}}{\dot{W}_{\text{shaft}}} \quad \text{(shaft-propeller efficiency)} \quad (17.10)$$

Notice that all engines consume more energy that what they supply: electrical engines, elastic engines, etc., but particularly thermal engines, where this internal energy conversion efficiency, $\eta_e$, is of the order of 30% to 50% (in electric motors it may be typically 95%).

Most propulsion systems may be considered in this way just power plants that apply a torque, $M$, to a shaft at a certain rotation speed, $\omega$, producing a power $\dot{W}_{\text{shaft}} = M\omega$. The shaft then is mechanically coupled to the propeller itself, which may be the friction wheels in land vehicles, or the pressure-reaction blade-propeller in sea and air vehicles.

But there are some propulsion systems based on the change of momentum of a working fluid, notably rockets, jet-engines and water jets, which have inlets and outlets through which some fluid flow. The momentum balance for an open system is (5.2):

$$\frac{d}{dt}(mv) = \vec{F}_v + \vec{F}_A + \sum_{\text{openings}} (\vec{v}_e \frac{dm}{dt} - p_e A_e \vec{n}_e) \quad (17.11)$$

where $m$ is the mass of the body being propelled (may be changing), $\vec{F}_v$ is the sum of externally applied volumetric forces (gravitational or electromagnetic), $\vec{F}_A$ the sum of externally applied surface forces at the wall, and $\vec{v}_e$ and $p_e$ the exit velocity and pressure through any opening of area $A_e$ with an outward normal vector $\vec{n}_e$.

Rocket propulsion

For steady horizontal motion of a body with just one exit (rocket) the momentum balance, in the direction of motion, reduces to (see Eq. 5.20):

$$T = m\dot{v}_e + (p_e - p_o)A_e \quad (17.12)$$
where $p_0$ is the pressure in the surrounding media (acting on the impermeable walls), that may be zero in outer space, and the propulsion efficiency, $\eta_p$, is defined as the ratio of thrust power, $Tv$, to whole power emanating from the body, that is the sum of thrust power, $Tv$ (communicated to the surrounding media) and residual kinetic power of the jet relative to the surrounding media, $\frac{1}{2}\dot{m}(v_e - v)^2$ (communicated to the jet); i.e.:

$$\eta_p = \frac{Tv}{Tv + \frac{1}{2}\dot{m}(v_e - v)^2} = \frac{\dot{m}v_e v}{\dot{m}v_e v + \frac{1}{2}\dot{m}(v_e - v)^2} = \frac{2}{v + \frac{v_e}{v_e}}$$

(17.13)

where the simplification of considering $p_e=p_0$ is always acceptable (it is true for subsonic exits, and for supersonic exits with 'adapted nozzle', and very approximate even for large pressure departures). Equation 17.13 is shown in Fig. 17.13.

![Fig. 17.13. Rocket propulsion efficiency vs. flight speed to exit speed ratio.](image)

Most rockets are heat engines because they transform the heating power of a fuel/oxidiser mixture into mechanical energy (the kinetic energy of the jet), and thus an energy efficiency may be defined by:

$$\eta_e = \frac{1}{2}\frac{\dot{m}v_e^2}{\dot{m}h_p}$$

(17.14)

where $h_p$ is the heating power of the fuel/oxidiser mixture per unit mass of reactants. Notice that high energy-efficiency demands high exit speed, demanding also high flight speed to keep reasonable propulsion efficiencies; i.e., rocket propulsion is inefficient at low speed. Compressed-gas rockets are also heat engines, but new magnetohydrodynamic and electrodynamic rockets are not.

Air-breathing propulsion

For steady horizontal motion of a body with one inlet and one exit (like a jet engine) the momentum balance, in the direction of motion, reduces to (see Eq. 5.20 and 17.12):

$$T = \dot{m}(v_e - v_i) + [(p_e - p_0)A_e - (p_i - p_0)A_i]$$

(17.15)

where the approximation to neglect the additional mass flow rate of fuel is introduced. The propulsion efficiency, defined as for rockets, becomes:
\[ \eta_p = \frac{\eta_p}{\eta_p - \eta_p} = \frac{\eta_p}{\eta_p - \eta_p} \]

(17.16)

where the approximation to consider the inlet speed equal to the flight speed has been introduced. Notice that the thrust, \( \Delta v \), may be obtained with a large \( \Delta v \) and small flow rate or vice versa, but the propulsion efficiency, \( \eta_p = 2/(2+\Delta v/v) \), shows that it is better to have a small \( \Delta v \) and a large \( \dot{m} \), what has been implemented in practice, where turbofans of high bypass have superseded simple turbojets.

Similarly to chemical rockets, air-breathing engines (turbojets) are heat engines that transform the heating power of a fuel into mechanical energy (the kinetic energy of the exiting jet minus the one at the inlet), and thus an energy efficiency may be defined by:

\[ \eta_e = \frac{1}{2} \frac{m v_e^2 - m v_i^2}{m \Delta h} \]

(17.17)

where \( h \) is the heating power of the fuel/oxidiser mixture per unit mass of fuel. Notice that the energy efficiency is based on thermal-to-mechanical conversion, and the propulsion efficiency is based on mechanical-to-propulsive ratio, thus, the efficiency using the fuel for propulsion is \( \eta_e \eta_p \). The overall energy balance of the air-breathing engine may help to identify every term; in axes fixed to the surroundings:

\[ \dot{H}_{chem} = \dot{W}_{prop} + \dot{K}_{jet} + \dot{H}_{jet} \]

(17.18)

with:

\[ \dot{H}_{chem} = \dot{m} h, \quad \dot{W}_{prop} = T v, \quad \dot{K}_{jet} = \frac{1}{2} \dot{m} (v_e - v_i)^2, \quad \dot{H}_{jet} = \dot{m} c_p (T_e - T_i) \]

(17.19)

\( \dot{H}_{chem} \) being the chemical enthalpy released, \( \dot{W}_{prop} \) the propulsive power, \( \dot{K}_{jet} \) the kinetic energy of the exit jet, and \( \dot{H}_{jet} \) the thermal enthalpy of the exit jet.

It is important to compare air-breathing propulsion with rocket propulsion for the same fuel expenditure (notice that for rockets \( \dot{m}_f = \dot{m} \), whereas for turbojets \( \dot{m}_f \) is of the order of 2% of \( \dot{m} \)). The quotient of respective thrusts is:

\[ \frac{T_{turbojet}}{T_{rocket}} = \frac{\dot{m}_f \left( v_{e,turbojet} - v_{i,turbojet} \right)}{\dot{m}_{fuel,rocket} v_{e,rocket}} = \frac{v_{e,rocket}}{v_{e,turbojet} + v} \]

(17.20)

showing that air-breathing engines are much better than rockets (if there is air!) because typical values are \( v_{e,turbojet} \sim 2 \cdot 10^2 \) m/s, and \( v_{e,rocket} \sim 5 \cdot 10^3 \) m/s.
COGENERATION

Cogeneration is the term coined for the combined (simultaneous) generation of work and heat (useful heat, really). Notice that, if a heat engine were perfectly designed, its necessary heat rejection to the environment would be at so close the ambient temperature that it would have no useful energy (i.e. no exergy); conversely, if a heating device were perfectly designed, its maximum temperature would be just above the desired utilization level. However, actual heat engines release waste heat at 100..400 K over ambient temperature, and boilers may use flames at 2000 K to heat water to 400 K. This practical poor design of either single (work or heat) producer is what justifies the interest in combined their production, although the saving in required components may be another advantage.

A handy cogeneration application is the cabin heating in a car, where engine heat rejection (from the exhaust or from the cooling fluid) is recovered when space heating is needed.

Cogeneration must serve two clients simultaneously, each with changing needs over time. Following the demand of a single client is already complex; e.g. to satisfy electrical power needs at home or in a firm, one sets the maximum capacity (and pays for this availability), and then make use of the power as needed (paying additionally for the energy consumed); the electricity-generation companies must also have a large-enough installed capacity (many power stations) and start-and-stop or modulate units as demand varies with time. Operation of a home or industrial boiler is similar: it must be modulated to follow the changing needs on sanitary hot water, space heating... If following the demand of a single client (power or heating) is already so cumbersome, satisfying the demand on time from several users is much more difficult, if not impossible, and in practice cogeneration just follows one demand (or none), i.e. it provides a more or less constant base power capacity, and additional power needs are covered by electricity from the grid, and by additional boilers.

Self-generation of electricity in industry is not the rule because the efficiency is larger in large power stations, even when accounting for up to 10% transmission losses; self-generation was only a redundant option for critical premises (e.g. hospitals, airports) in case of grid failure. Thermal power needs, however, are always in-situ generated because of the high transport costs. But the synergy of using in-situ equipment to produce both heat and work can be great, and it is no wonder that cogeneration systems were implemented in large-consumer premises (department stores, primary industry, large vehicles...) since the early 1900s.

Cogeneration is provided by using a heat engine (a steam turbine, a gas turbine or a reciprocating engine) to generate work and, at the same time, heat (usually in a exhaust gasses boiler). The great advantage of cogeneration is the energy saving; an additional advantage may be the autonomy gained by self-production, and the main drawback is that the ratio of work-to-heat generation is rather stiff, none can be easily accumulated, and the actual need of work and heat may vary a lot with time (that was one of the main advantages of central production: the levelling of the averaged demand). To stiffness of a cogeneration plant is relaxed by the possibility to send excess work-power to the electrical grid, a convenience that has been enforce by public authorities on account of the social benefit that saving in
primary energy resources (fossil fuels) cogeneration brings. According to the prime mover, typical
cogeneration systems are:

- **Reciprocating engine cogeneration.** The useful heat-to-work ratio is around 0.6. It is only used for
direct heating with the exhaust gases or the cooling water, but not to generate vapour.

- **Gas turbine cogeneration.** The useful heat-to-work ratio is around 2. It does not require changes in
the power plant, it is a versatile cogeneration system, and its use is expanding in most industries:
textile, chemical, food. Because the exhaust gases are relatively hot (some 700 K), it is suitable
for high temperature applications.

- **Vapour turbine cogeneration.** The useful heat-to-work ratio is around 7 (suitable for the iron
industry, but still low for the ceramic and glass industry). It directly provides useful vapour, either
by extracting some of it at an intermediate stage in the turbine, or by not expanding to the low
pressures of ambient temperatures but to higher than atmospheric pressures (it is called a back-
pressure turbine), to render the whole turbine outlet vapour useful.

Efficiencies of cogeneration plants are high, but care must be paid not to mix work and heat values
(everybody knows that work is usually two or three times more expensive than heat, i.e. the output should
be consistently measured in exergy, so that the, instead of \( W + Q \), the output value is \( W + Q \left( 1 - T_0 / T \right) \), where \( T \) is the temperature at which heat is delivered.

Although cogeneration usually refers to the combined production of work (electricity) and heat (often
abbreviated to CHP, combined heating and power), it can also be applied to work and cold, or the
combination of work, heat, and cold (sometimes termed 'trigeneration', or CCHP, combined cooling
heating and power), or even to the combined production of some useful substance (e.g. desalinated water,
or synthetic fuel) and some energy service (power, heating, or cooling).

Goswami cycle (still at research stage; proposed by Dr. Goswami in 1998) is a novel thermodynamic
cycle that uses a vapour binary mixture to produce power and refrigeration simultaneously (it is a
combination of a Rankine power cycle and an absorption cooling cycle).

Trigeneration systems are usually based on a natural-gas-fuelled engine (a gas turbine most of the times),
with an electrical generator, and an absorption heat pump (see Refrigeration) that works as a refrigerator
in summer (or as a chiller, in general), and as a heating pump in winter (with a natural-gas burner to
adjust the load).

As said before for cogeneration, a big problem in trigeneration is that the equipment can only work for a
nearly fix share of work, heat, and cold, whereas the actual demand may show wide changes with time in
the share and in total amount of power.

**EFFICIENCY IN POWER GENERATION WITH HEAT ENGINES AND OTHER GENERATORS**

Power cannot be generated; only converted from one form to another; what is implied is the generation of
some useful power at the expense of some other less-convenient power. But recall that we have restricted
here the term ‘power’ to mechanical or electrical power (i.e. excluding thermal power), so that power
generation can be seen as the generation of electricity (or equivalent power) from any other energy source: thermal, chemical, nuclear, etc.

Power generation efficiency can be defined as “useful output power divided by input power”, but it is not rigorous enough since at least two choices exist for the evaluation of input power: \( a) \) heat-equivalent power, and \( b) \) work-equivalent power. Table 2 presents typical values of power generation efficiencies using the raw input power (choice \( a) \), the most commonly used, although the net input power criterion, i.e. the exergy or available energy of the raw energy source, would give a sounder measure of the ‘technological efficiency’ of the power plant.

<table>
<thead>
<tr>
<th>Energy source</th>
<th>Typical efficiency [%]</th>
<th>Typical range [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photovoltaic</td>
<td>10</td>
<td>5..15</td>
</tr>
<tr>
<td>Solar thermal</td>
<td>15</td>
<td>10..25</td>
</tr>
<tr>
<td>Gas turbine</td>
<td>30</td>
<td>15..38</td>
</tr>
<tr>
<td>Spark Ignition ICE</td>
<td>30</td>
<td>25..35</td>
</tr>
<tr>
<td>Nuclear</td>
<td>33</td>
<td>32..35</td>
</tr>
<tr>
<td>Steam turbine</td>
<td>33</td>
<td>25..39</td>
</tr>
<tr>
<td>Wind turbine</td>
<td>40</td>
<td>30..50</td>
</tr>
<tr>
<td>Compression Ignition ICE</td>
<td>40</td>
<td>35..49</td>
</tr>
<tr>
<td>Fuel cell</td>
<td>45</td>
<td>40..70</td>
</tr>
<tr>
<td>Combined GT-ST</td>
<td>50</td>
<td>45..60</td>
</tr>
<tr>
<td>Hydroelectrical</td>
<td>85</td>
<td>70..90</td>
</tr>
</tbody>
</table>

It is without saying that power generation efficiency is not the only criterion, neither the most important, to quantify engine excellence. The generic goal of maximum power at minimum cost, should include the cost of design (e.g. new technologies), cost of manufacture (e.g. new materials), cost of implementation (e.g. size and weight), cost of operation (e.g. specific fuel consumption, but also pollutant emissions and noise level), cost of maintenance (reliability), and even the cost for disposal.

**TYPE OF PROBLEMS**

Besides housekeeping problems of how to deduce one particular equation from others the types of problems in this chapter are:
1. Solve Otto, Diesel and mix cycles.
2. Solve gas turbine cycles (Brayton): simple, regenerative, combined, cogeneration.
3. Solve steam turbine cycles (Rankine): simple, regenerative, combined, cogeneration.
4. Solve propulsion-power problems.

Working fluid properties are usually approximated with the perfect substance model (PSM)), i.e. the perfect gas model for gases and vapour, the perfect liquid model for liquids, and Clapeyron's equation (or better Antoine's correlation) to liquid-vapour equilibrium. Of course, PSM cannot be used close to the critical point of the working fluid, where detailed data is used (from dedicated charts, or numerical codes). Isentropic efficiencies in turbo-machinery, and temperature jumps in heat exchangers, are the most common modification introduced to ideal cycles.